Dynamics, Numerics, and Plain Old Ignorance; progress and problems in numerical ice sheet/stream/shelf modeling

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Why the silly title?

- "plain old ignorance" is not an insult to anyone!
- but the biggest difficulties in getting good numerical ice sheet models have *nothing to do with*
  - continuum mechanics
  - numerical analysis
  - coupling of models
  - lack of computing power
- glaciologists and inverse ice modelers sometimes observe that: *not enough is known to build predictive ice sheet models*
- we cannot ignor this point of view

What are the biggest difficulties?

here is a modest wishlist for 4 simple devices which could fix the biggest difficulties in building predictive ice sheet models . . .

Wishlist item 1: Measuring three dimensional fields

- temperature within the ice very important (ice sheets close to melting point)
- in this regard, ice flow more like forging iron than air/water fluid flow

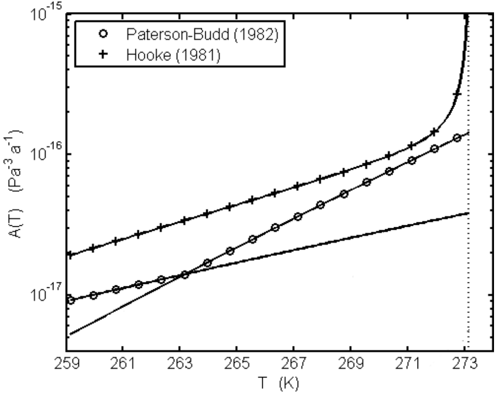
so, can we have one of these that can be cheaply towed through the ice to record the temperature in a whole bunch of places?

thermometer on a string

fast drill? *temperature at depth through remote sensing?*

Comment: Temperature dependence of ice flow

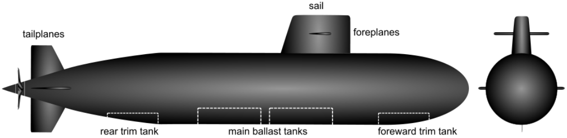
viscosity is *very* sensitive to temperature:



Wishlist item 2: Observing the base of the ice

- base of flowing ice sheets and streams is hard to observe
- sea ice modeling is also an important problem, too . . .
- *isn't it nice to have views from above and below*?

can we have one of these, designed to go under a land ice sheet and look up?:



For now we have inverse modeling . . .

- since we lack towable ice thermometers and crustal submarines we're stuck with inverse models
- inverse modeling exploits (relatively) rich *boundary observations at the current instant*
- example: "ice sheet model spinup" inverts measured, current surface conditions for the temperature field at depth (added: time dimension from point locations = ice core records)
- principled inverse modeling example: estimate basal shear stress in ice streams with vertically-integrated models (MacAyeal/Joughin/Gudmundsson/Sergienko)
- another: flowline/crossflow Stokes models for glaciers (Gudmundsson/Truffer)
- also: invert radar isochrones to get velocity (Eisen)

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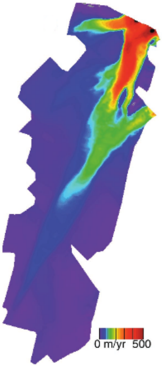
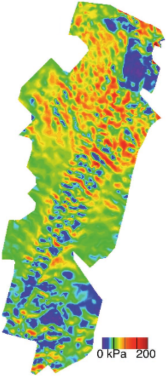
Inverse modeling example



observed velocity of NE Greenland ice stream



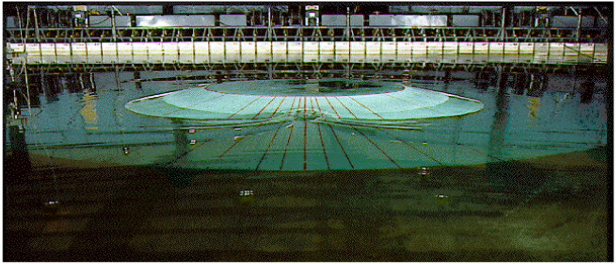
estimated basal shear stress

figures from Joughin et al 2001 JGR

Wishlist item 3: Full-scale laboratory reproductions

- one of the hardest numerical aspects for ice sheet flow is margin movement

wouldn't it be nice to validate numerical ice sheet models with something like this:



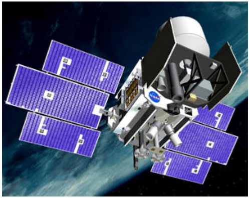
wavetank for simulating tsunami runup on a conical island

take Silly Putty seriously for validation? *. . . PISM can do weird flow laws (maybe)*

Wishlist item 4: More than one snapshot in time

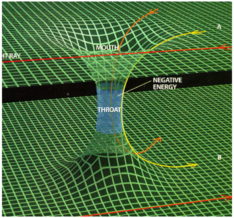
- $\pi \times 10^7$
- so 50 years of glaciology has yielded just one snapshot at one instant, really
- [*Ed*: anecdote of the NPR hurricane modeler, "does warming climate affect hurricane frequency and intensity?" w response ". . . only have thirty years of satellite imagery . . ."]

so, can someone please combine these technologies and generate hi res video of ice sheet dynamics for (say) the last four ice age cycles?:



ICESAT

+



wormhole

An early conclusion

- these are obviously essential:
  - basic glaciological research
  - cryosphere remote sensing
- they justifiably more expensive than numerical modeling!

Ice sheets, streams, & shelves: basic facts

- ice sheet/stream/shelf flow, *unlike* ocean and atmosphere, is slow, meaning balance of momentum is very well approximated by balance of non-inertial stresses
- *like* atmosphere and ocean, ice sheets/streams/shelves are shallow (whether or not models take advantage . . .)
- ice flow, *like* ocean, is incompressible to good approximation (below layer of 10 s of meters of compacting firn; most important for ice shelves?)
- upper surface of ice is always free to move, so finding domain of flow is part of the problem

Mass continuity for ice sheets/streams/shelves

- let $z = h(t, x, y)$ be ice surface, (u, v, w) be velocity, and $\mathbf{U} = (u, v)$ be horizontal velocity
- surface kinematical:

$$\frac{\partial h}{\partial t} = a + w - \nabla h \cdot \mathbf{U}$$

- incompressibility allows rewrite as "map-plane continuity":

$$\frac{\partial H}{\partial t} = a - \nabla \cdot \left(H\, \overline{\mathbf{U}}\right)$$

  where H is thickness and $\overline{\mathbf{U}}$ is vertically-averaged hor. vel.
- these equations apply whether the model is shallow or not
- and they hold regardless of dynamical model (ice shelves, with sliding, any flow law, membrane stresses, full Stokes, ...)

Non-sliding ice sheets are *diffusive*

- in nonsliding shallow case one can include dynamics into mass continuity to get simplest form:

$$\frac{\partial H}{\partial t} = a + \nabla \cdot (D\, \nabla h)$$

- we have a "Fourier's" law for ice sheets:

  nonsliding shallow ice flows opposite the surface gradient

  or

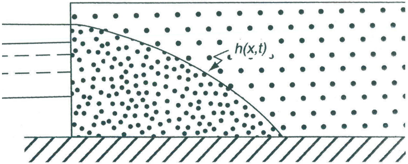
$$\mathbf{Q} = H\, \overline{\mathbf{U}} = -D\, \nabla h$$

- *diffusivity* $D = D(T, H, |\nabla h|)$ can go to zero
- this is like the porous medium equation ...

Compare to porous media

- porous medium equation is a *degenerate nonlinear diffusion* for the free upper surface of the saturated part

$$\partial_t h = \nabla \cdot (m h^{m-1} \nabla h) = \triangle(h^m)$$



Ice flow is *doubly nonlinear* and *doubly degenerate*

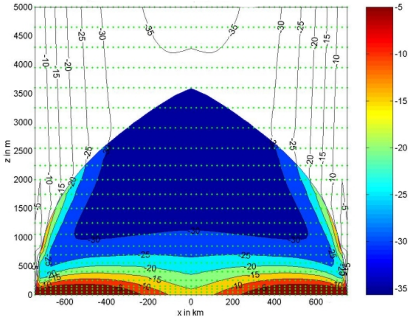
- diffusive if nonsliding and shallow; this includes thermocoupled case
- combined thermocoupled equation is *roughly*:

$$\frac{\partial H}{\partial t} = a + \nabla \cdot \left(\mathcal{A}(T) H^{n+2} |\nabla h|^{n-1} \nabla h\right)$$

  where $\mathcal{A}(T)$ is a generalized softness which comes from vertical integration
- as noted, diffusivity degenerates two different ways (because $n > 1$)

Does thermocoupling change anything? Yes!

suppose you work hard and model thermocoupled nonsliding ice
. . . you get something like this:



- just a thin layer of soft ice underneath a thick strong cap
- has a boundary layer of temperate ice (see Fowler)
- temperate ice "squirts" out from underneath strong cap; strain heating then feeds back to keep it squirting
- are there easier fluid analogs of this?

Compare to Hele-Shaw

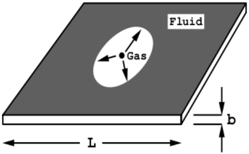
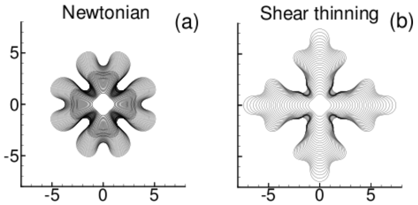


Figure 1: Hele-Shaw cell



Hele-Shaw flow . . . *which can be unstable*

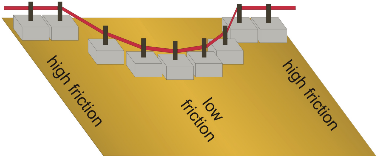
figures from L. Kondic et al 1998 *Phys. Rev. Letters*; comparison suggested to me by C. Schoof

- Hele-Shaw cells show Taylor fluid instability
- thermomechanically coupled shallow ice shows analogous fundamental kind of instability (discovered by T. Payne; new to fluid science?)

Elastic-with-coulomb-friction analog of an ice stream

C. Schoof's (2006 J. Fluid Mech.) analog of ice streams is *a mesh of blocks on a slab, connected by springs*

- each block sticks or slips according to a Coulomb friction law
- "springs" are analogs of the membrane (longitudinal) stresses in ice streams
- which blocks slide and which do not is solved simultaneously with finding the strains (analog: strain rates in ice streams)



- this analogy extends the dragging shelf model of MacAyeal
- Schoof offers a well-posed free boundary problem for ice streams
- PISM implements it (among other models; below)

Fluid analog for ice shelves

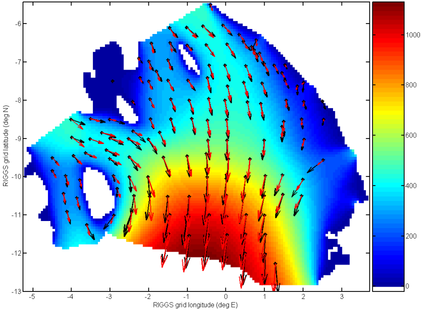
- ice shelves analogous to very viscous oil floating on water
- an early success for ice flow modelers . . .

Validation of an ice shelf flow model

color shows PISM's modeled speed (m/a) on Ross Ice Shelf with 6.8 km grid

black arrows are observed velocities (RIGGS 1983)

red arrows are PISM model velocities



- typical result from at least five diagnostic ice shelf models in mid 90s (MacAyeal et al 1996)
- 6.8 km grid is fine enough to resolve geometry and ice stream/glacier inputs

Summary: fluid comparisons for ice flow

ice sheet/stream/shelf flow is like:

1. flow through porous medium
2. fluid squirting from beneath rigid top (Hele-Shaw)
3. sticky blocks on a slab
4. viscous oil on water

conclusions:

- numerical models for whole ice sheets must handle ice versions of each of these ice flow behaviors
- ideally in a unified continuum model (but PISM is not there yet . . .)

recall the parable of the blind men and the elephant?

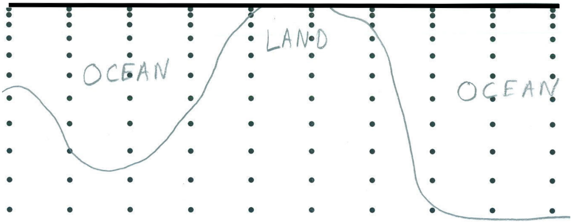
Numerics comparison: a global ocean circulation model

some of you can call to mind the equations of an ocean circulation model:

(*imagine a lot of equations here!*)
(*don't worry; I won't list them*)

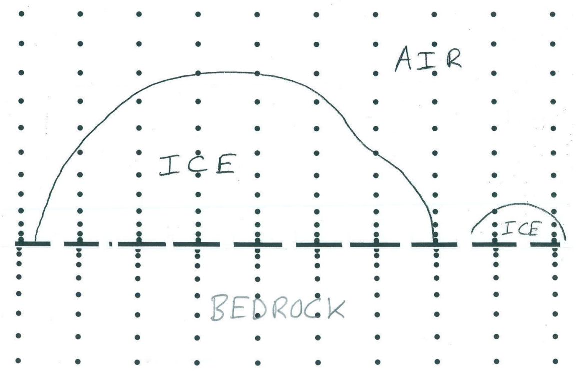
- ocean is shallow so models are shallow
- interaction with atmosphere (wind force, evaporation) means top layers are most interesting
- model is mostly *hyperbolic*; e.g. leapfrog time-stepping
- has diffusion terms coming from parameterized "sub-grid phenomena" (turbulent mixing)
- advection (transport) generally dominates diffusion

Grid for CCSM POP



- CCSM = Community Climate System Model
- POP = Parallel Ocean Program (= CCSM ocean component)

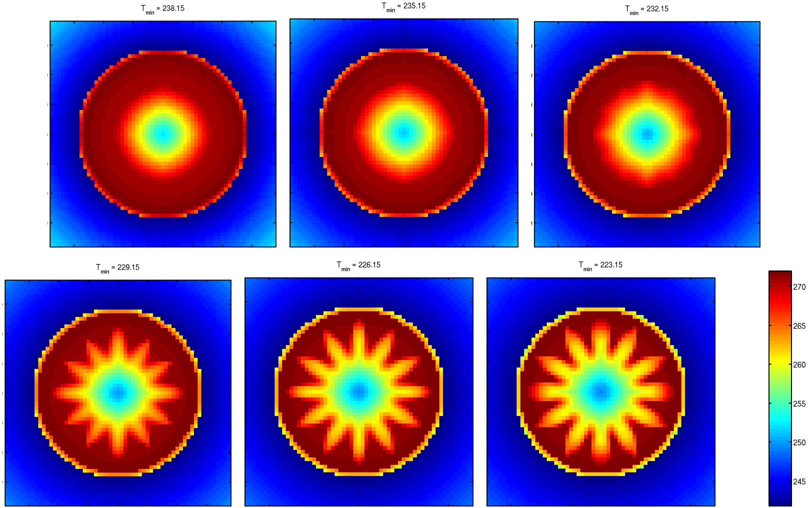
Grid for an ice sheet model (PISM)



- most ice sheet models do a "σ-transformation"
- PISM (ice sheet model) and POP (ocean circulation model) do not

What's going on here?

color shows model basal temperature for simplified geometry
thermomechanically-coupled, non-sliding shallow ice approximation.



Intercomparison

- intercomparison exercises help modelers meet and understand various possibilities
- earlier: EISMINT I, EISMINT II, EISMINT-Greenland, EISMINT-Ross
- just completed: ISMIP-HOM, ISMIP-HEINO
- current: MISMIP
- intercomparison has identified points of communal confusion; spokes above are an example
- but intercomparison doesn't measure error

Certainty (for once!)

- there are nontrivial exact solutions to the ice flow equations for each of these cases
  - isothermal nonsliding shallow ice (Halfar 1983; Bueler et al 2005)
  - thermocoupled nonsliding shallow ice (Bueler et al 2005)
  - ice shelves
  - plastic till ice stream free boundary problem (Schoof 2006)
- verification-by-exact-solution can be built into ice sheet models to increase user/developer confidence; PISM has this
- verification can say how fine a grid has to be for desired accuracy

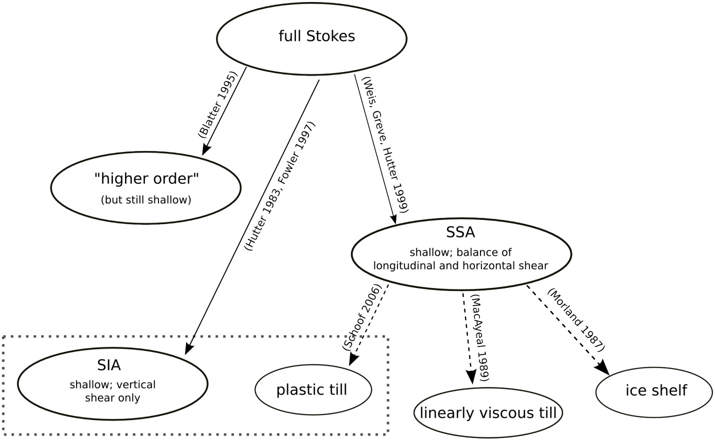
PISM = Parallel Ice Sheet Model

- open source; documentation and source code at
  www.pism-docs.org
- 75 page *User's Manual* includes how to perform verification (lots), EISMINT II, EISMINT-Ross, and EISMINT-Greenland
- started 2003; under active development
- structurally parallel using PETSc and MPI; has run on 500 processors
- solves shallow sheet/stream/shelf stress balances all in parallel

PDEs approximated by PISM

- map-plane conservation of mass
- incompressibility
- shallow approximation of conservation of energy
- bedrock thermal model
- computation of basal melt or freeze-on from conservation of energy
- model for till yield stress from stored water in till (below)
- earth deformation (by new fast method)

Stress balances



PISM can solve each of the bottom row of models

(rectangle shows model for next experiment . . .)

Preferred "sliding law" is ice stream equations

- traditional sliding laws have sliding velocity a function of driving shear stress, switched on when base reaches pressure-melting temperature
- Schoof's plastic till ice stream model as "sliding law" a free boundary problem
- emergent ice streams
- sliding flow is controlled by balance of "membrane" stresses
- vertical plane shear from (SIA) still present; it is added back; next

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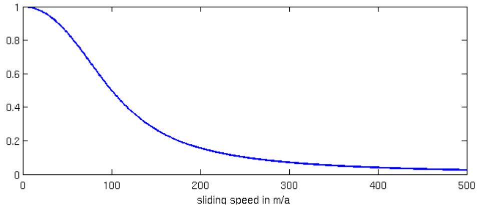
Heuristic: combine sliding with shear

$$\mathbf{U} = f(|\mathbf{v}|)\,\mathbf{u} + \mathbf{v}, \qquad (1)$$

- $\mathbf{v}$ = ice stream/shelf hor. velocity for given geometry
- $\mathbf{u}$ = nonsliding shallow ice approx. hor. vel. for given geometry
- $\mathbf{U}$ is used in incompressibility, temperature, and mass continuity equations
- not merely "heuristic" in majority of ice sheet
- no replacement for Stokes or Blatter, but cheaper to compute
- same heuristic as in D. Pollard flowline model

example $f()$:



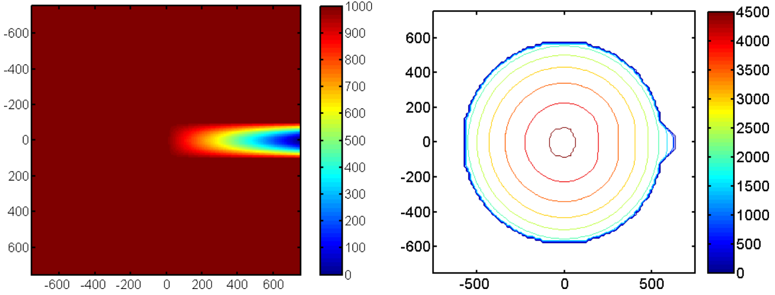
On time-stepping

- explicit
- diffusive vertical-plane-shear flow from SIA has a condition for stability (*typical: 0.3 year for 12.5 km grid*)
- contribution to mass cont. from sliding is upwinded; recall

$$\frac{\partial H}{\partial t} = a - \nabla \cdot \left(H\overline{\mathbf{U}}\right)$$

- ...which gives another condition for stability (*typical: 0.1 year for 12.5 km grid*)
- plastic till model used as sliding law everywhere at base
- temperature equation solved every time step; *dirt cheap compared to SSA solution*

Experiment starts with shallow ice approximation

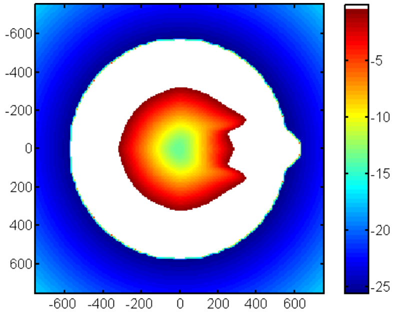


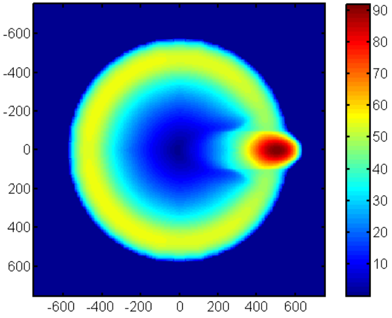
Left: bed elevation (m) *Right*: surface elevation (m) at end of 200ka run

ice flow model is non-sliding thermocoupled shallow ice approximation (SIA)

this is EISMINT II experiment I

EISMINT2 experiment I, cont.



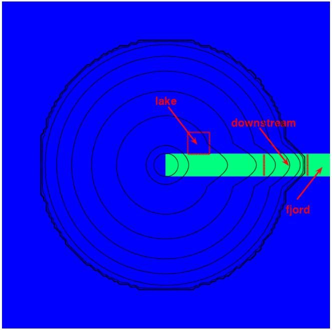


Left: homologous basal temperature (°C) at end of 200ka run; white areas at pressure-melting

Right: vertically-integrated horizontal speed (m/a) at end of run

there "ought" to be an ice stream but it hasn't happened

Add plastic till ...



green strip has till friction angle $\phi = 5^\circ$

blue area has $\phi = 20^\circ$

contours show ice thickness

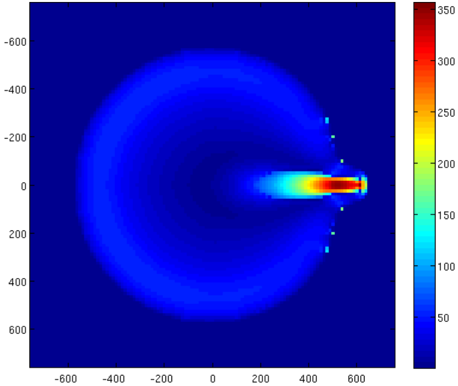
("lake", "downstream", and "fjord" apply to parameter study experiments; later)

till yield stress τ_c is computed by $\boxed{\tau_c = (\tan\phi)(\rho g H - p_w)}$ (e.g. Paterson)

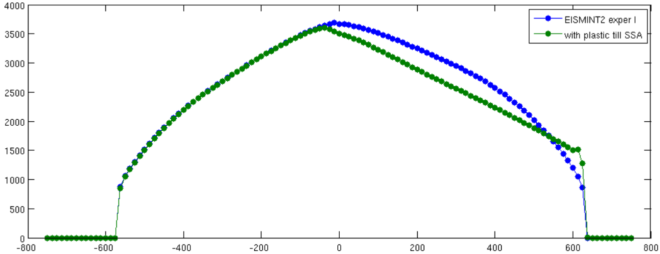
p_w = pore water pressure in till (equal to 0.95 of overburden when till fully saturated, but $= 0$ when base is frozen)

Result from a 5000 model year run

use end of EISMINT II experiment I as starting state, run for 5000 model years on 12.5km grid, and look at vertically-averaged horizontal velocity:

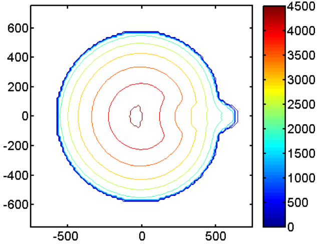


ice thickness

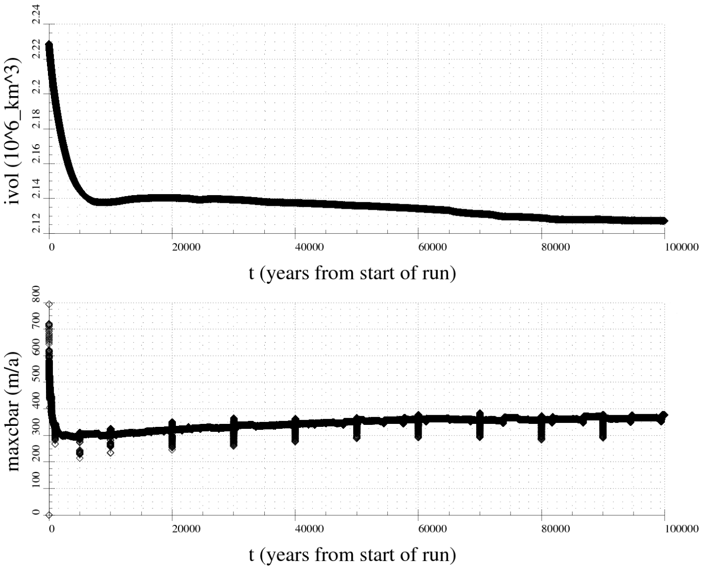


Above: ice thickness along centerline of trough (x-axis)

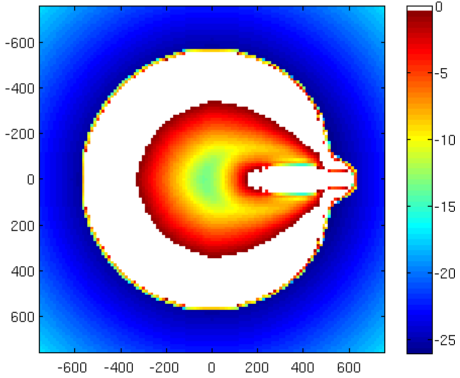
Right: surface elevation contours for plastic till SSA version

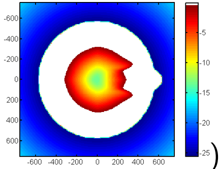


Longer run: 100ka



basal homologous temperature (°C) at 5ka



(compare )

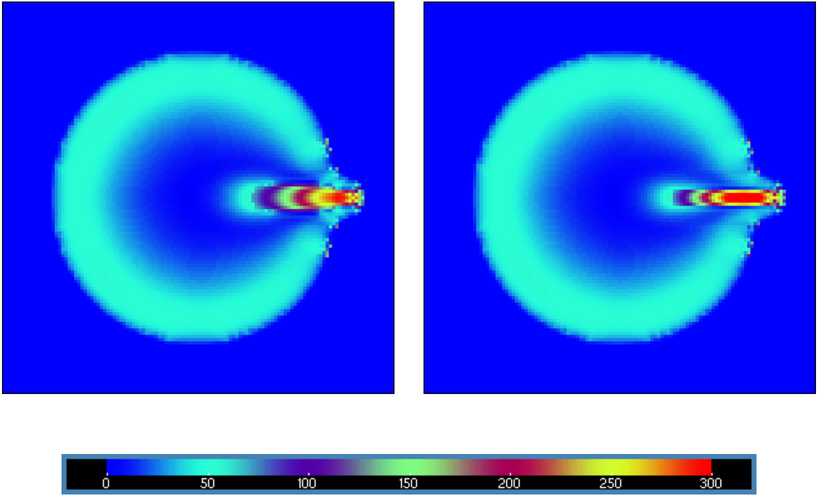
Experiments: other parameter choices

Experiment	Grid (hor,vert)	Duration	Modification to P0
P0	(12.5 km, 20 m)	5k a	
P1	(12.5 km, 20 m)	5k a	*no trough*: flat bed
P2	(12.5 km, 20 m)	5k a	*narrower strip*: 50 km wide weak till strip
P3	(12.5 km, 20 m)	5k a	*stronger downstream till*: $\phi = 8°$ in "downstream" part of strip
P4	(12.5 km, 20 m)	5k a	*lake*: $\phi = 0°$ in "lake" area
P5	(12.5 km, 20 m)	5k a	*fjord*: $\phi = 0°$ in "fjord" area
P6	(25 km , 20 m)	5k a	
P7	(7.5 km , 20 m)	5k a	
P8	(5 km , 20 m)	5k a	
P9	(12.5 km, 10 m)	5k a	
P10	(12.5 km, 5 m)	5k a	
P0cont	(12.5 km, 20 m)	100k a	

experiments start with final state of EISMINT II experiment I (except P1 starts from end of exper A)

weak till strip widths of 100 km (P0) and 50 km (P2)

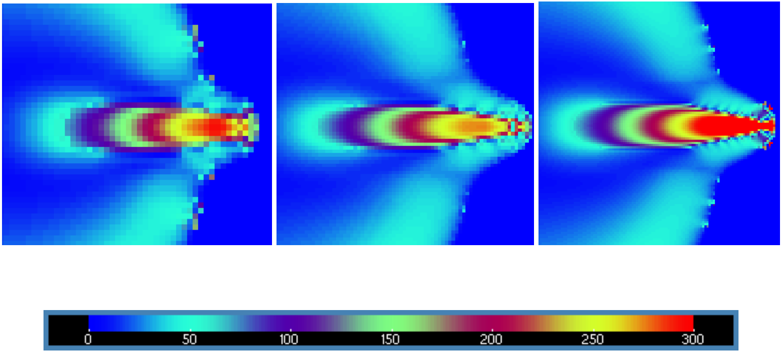
snapshot at 5k model years after start of sliding:



common velocity scale (m/a)

ice stream detail: 12.5 km, 7.5km, and 5km grids

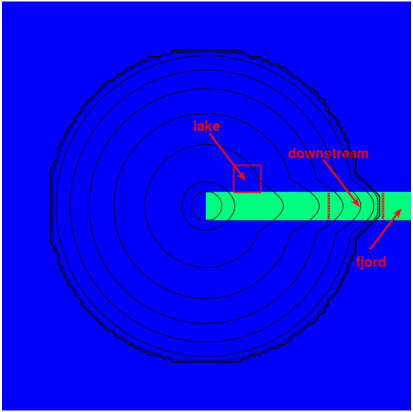
look at vertically-integrated horizontal velocity in ice stream region for experiments P0, P7, and P8:



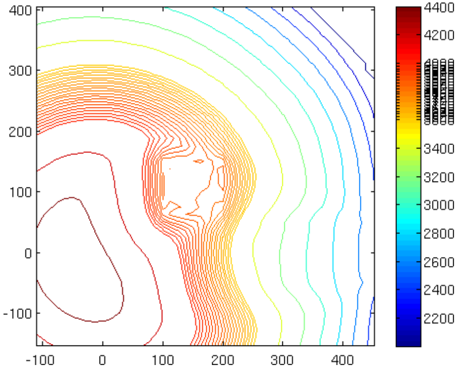
common velocity scale (m/a)

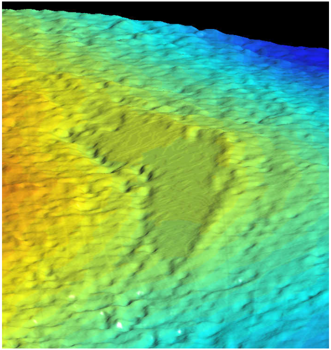
- each shown at 5k model years after start of sliding
- weak till strip is 100km wide (but the ice stream narrows dynamically!)

Experiments P4 and P5 add “lake” and “fjord”



"Lake" result: surface elevation

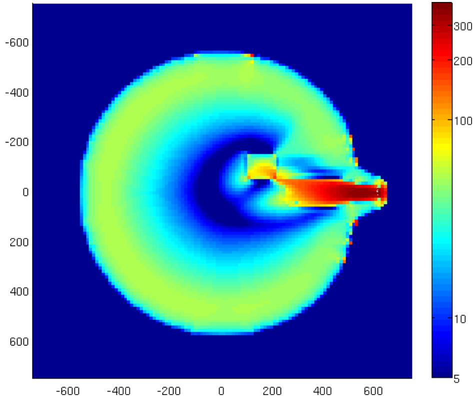


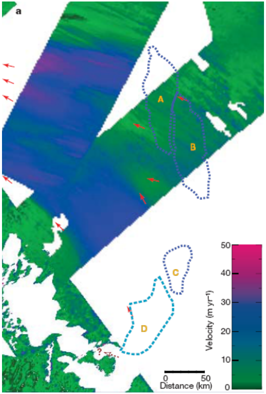


Left: Detail of surface topography above $\theta = 0^\circ$ till "lake" in experiment P4. Contours every 20 m between 3600 m and 4000 m.

Right: a perspective view of the ice surface above Lake Vostok compiled from ERS-1 radar altimeter data provided by the National Snow and Ice Data Center. Lake Vostok is the flat, featureless area. From Michael Studinger's web page.

"Lake" result: vertically-averaged horizontal speed

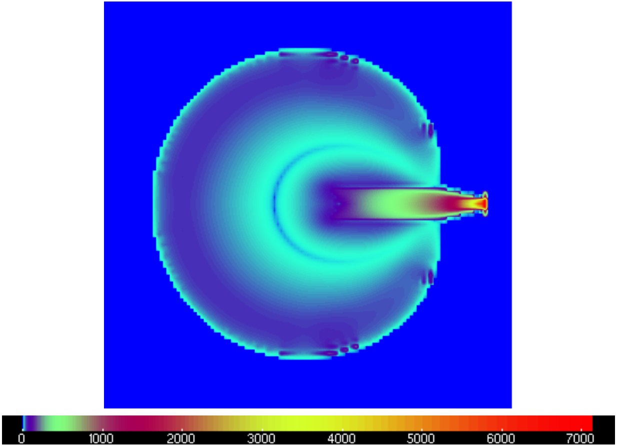




Left: Vertically-averaged horizontal speed for P4 at 5ka.

Right: InSAR relative ice-surface velocity in m/a for the Recovery ice stream catchment. Red arrows show locations of clearly defined flow stripes. From Belle et al (2007) "Large subglacial lakes in East Antarctica at the onset of fast-flowing ice streams," *Nature*.

"Fjord" result



Conclusions

- modeling no substitute for better observations
- ice sheet modelers can learn from other geophysical flows
- teams are essential: glaciologists, math/CS people, and the GCM-aware
- ice sheet modeling means obligatory inverse modeling; inverse people must be part of team
- *shallow models will be with us forever*; nirvana is knowing how to choose and how to couple
- verification through exact solutions helps with designing and maintaining ice sheet models
- . . . while intercomparison identifies weaknesses of models
- PISM is open, flexible, scalable: www.pism-docs.org