Ice Sheet Modeling
Numerics and Visualization

Ben Sperisen

Who I am and what I did here

- Majoring in physics, computer science, and economics at New Mexico State University.
  - This will be my senior year.
- Worked on ice sheet modeling:
  - learned about how an ice sheet model works
  - improved the way scientists can look at their model's results

What my project is about

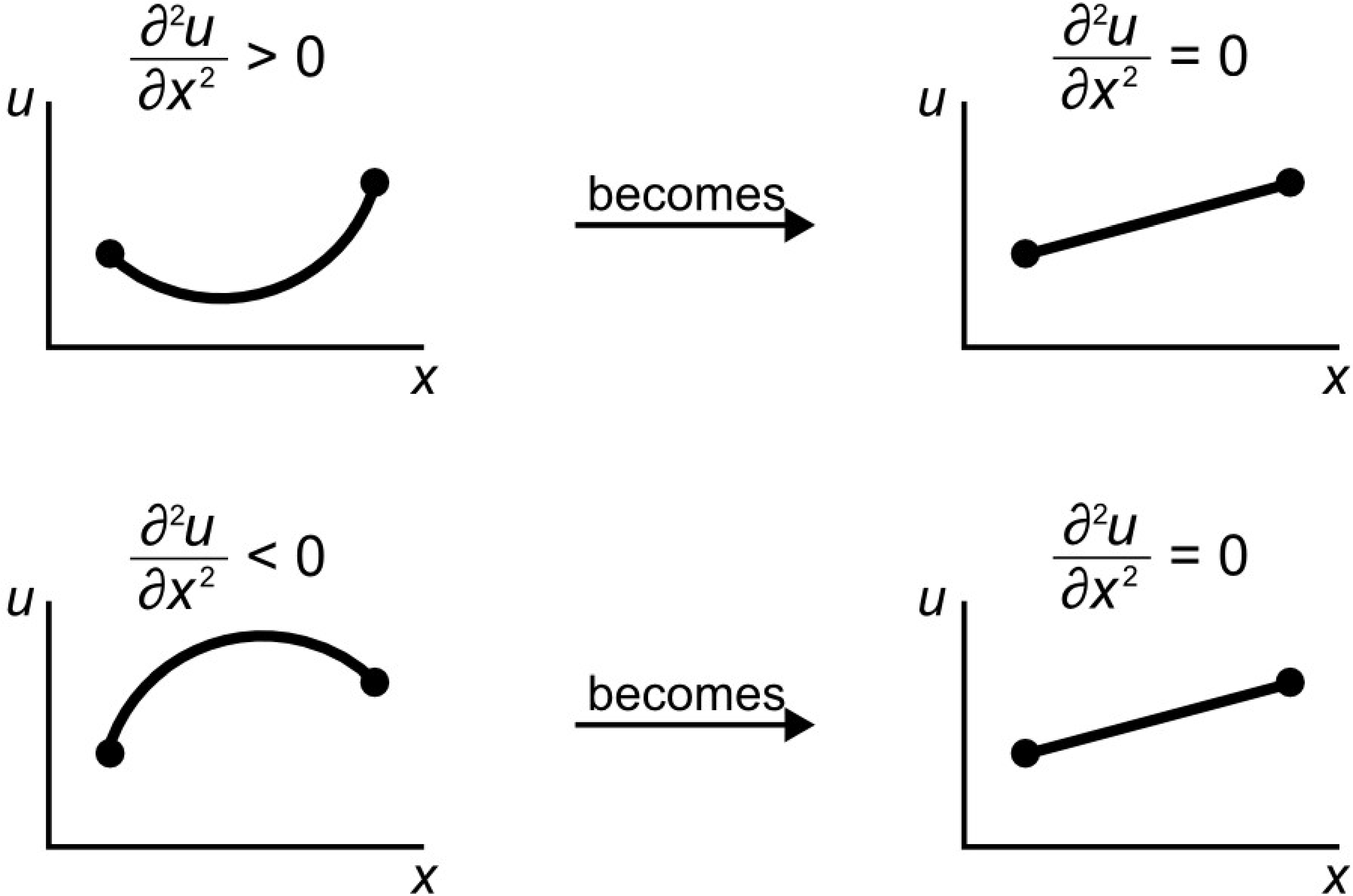
- How the equations for ice sheets are solved
  - The Finite Difference Method
- Improving Visualization of the Results of PISM, a Parallel Ice Sheet Model
  - Modifying the output files so they work in IDV
  - Taking advantage of animations and 3D plots to understand results of the model

PDEs describe lots of things ...

- Partial Differential Equations (PDEs) are used to describe a large variety of phenomena, including
  - electric and magnetic fields,
  - heat propagation,
  - fluid flow,
    - air over airplane's wings
    - water flowing in an ocean
  - car traffic,
  - and ice sheets (a fluid flow problem).

The Heat Equation

- Simple example: $$\frac{\partial u}{\partial t} = \alpha \nabla^2 u = \alpha \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$
  - $u(x,y)$ is temperature
  - α is some constant
- What does this mean?
  - The more bumpy temperature is, the faster it smooths out.



$\frac{\partial^2 u}{\partial x^2} > 0$
u
x
becomes
$\frac{\partial^2 u}{\partial x^2} = 0$
u
x
$\frac{\partial^2 u}{\partial x^2} < 0$
u
x
becomes
$\frac{\partial^2 u}{\partial x^2} = 0$
u
x

Solving the Heat Equation

- Using the finite difference method, we get this numerical solution:

$$u_{i,j,k+1} = \frac{\Delta t}{(\Delta x)^2}\left(u_{i+1,j,k} + u_{i-1,j,k} + u_{i,j+1,k} + u_{i,j-1,k}\right) + \left(1 - 4\frac{\Delta t}{(\Delta x)^2}\right)u_{ijk}$$

- What does this mean?
  - The temperature at a point is updated with a weighted average of the temperatures of its neighbors and itself.

When a numerical solution blows up

- What if $\frac{\Delta t}{(\Delta x)^2} > \frac{1}{4}$?
-
  - The coefficients for the neighbors' temperatures add up to *more* than 1, and
  - the coefficient for u_{ijk} is *negative*.
- This is not a weighted average anymore.
  - The heat flowing out of a point is more than the point actually has.
  - The solution blows up.

The Ice-Sheet Equation

- The ice sheet equation is more complicated:

$$\frac{\partial H}{\partial t} = M + \nabla \cdot \left(\Gamma H^{n+2} |\nabla H|^{n-1} \nabla H \right)$$

  - H is thickness (height on flat bedrock)
  - M is accumulation (snowfall)
  - Γ is some constant
  - n is some exponent in the range $1.8 \leq n \leq 4$ (3 is usual pick)
- What does this mean?
  - Ice flows downhill, and it flows fastest where the ice is thick and steep (and ice gets thicker when snow falls on it), like molasses on a plate.

Stability of Ice-Sheet Solution

- The requirement for stability of the solution is

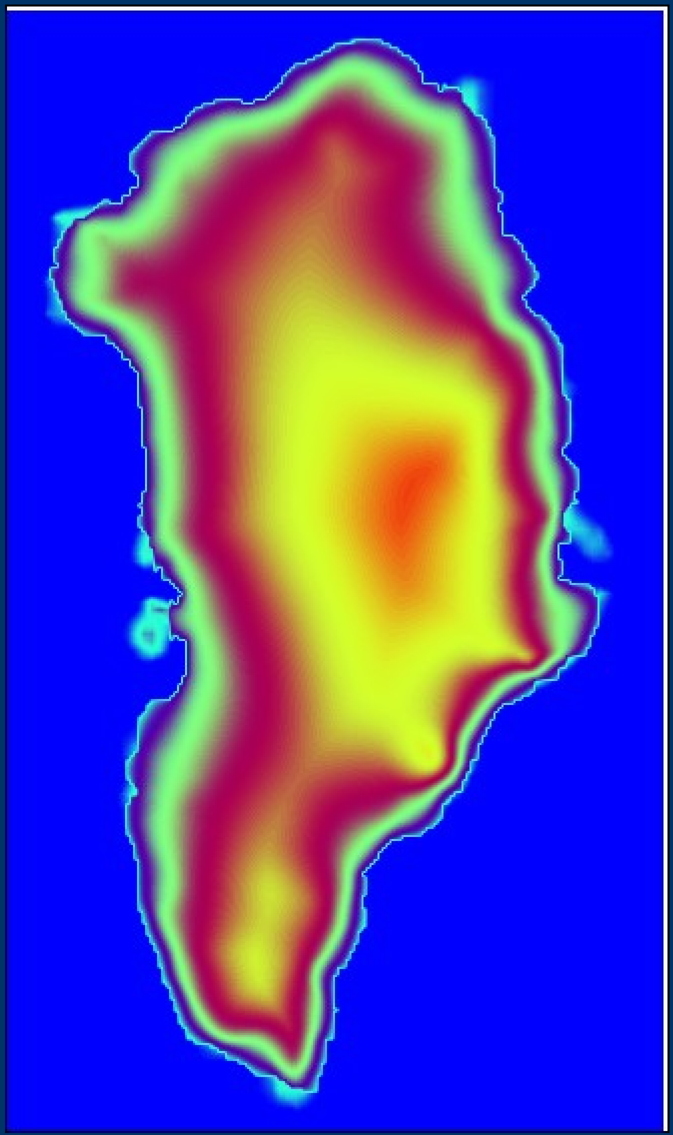
$$\frac{\Delta t}{(\Delta x)^2} < \frac{1}{6} \max \left(\Gamma H^{n+2} |\nabla H|^{n-1} \right)$$

- This means we can vary the time step as needed to improve performance.

Visualization

- PISM outputs a NetCDF file at the end of a run containing many variables, including:
  - ice thickness,
  - speed of ice,
  - temperature, and
  - age.
- Previously, visualization was done primarily using ncview.

ncview showing ice thickness



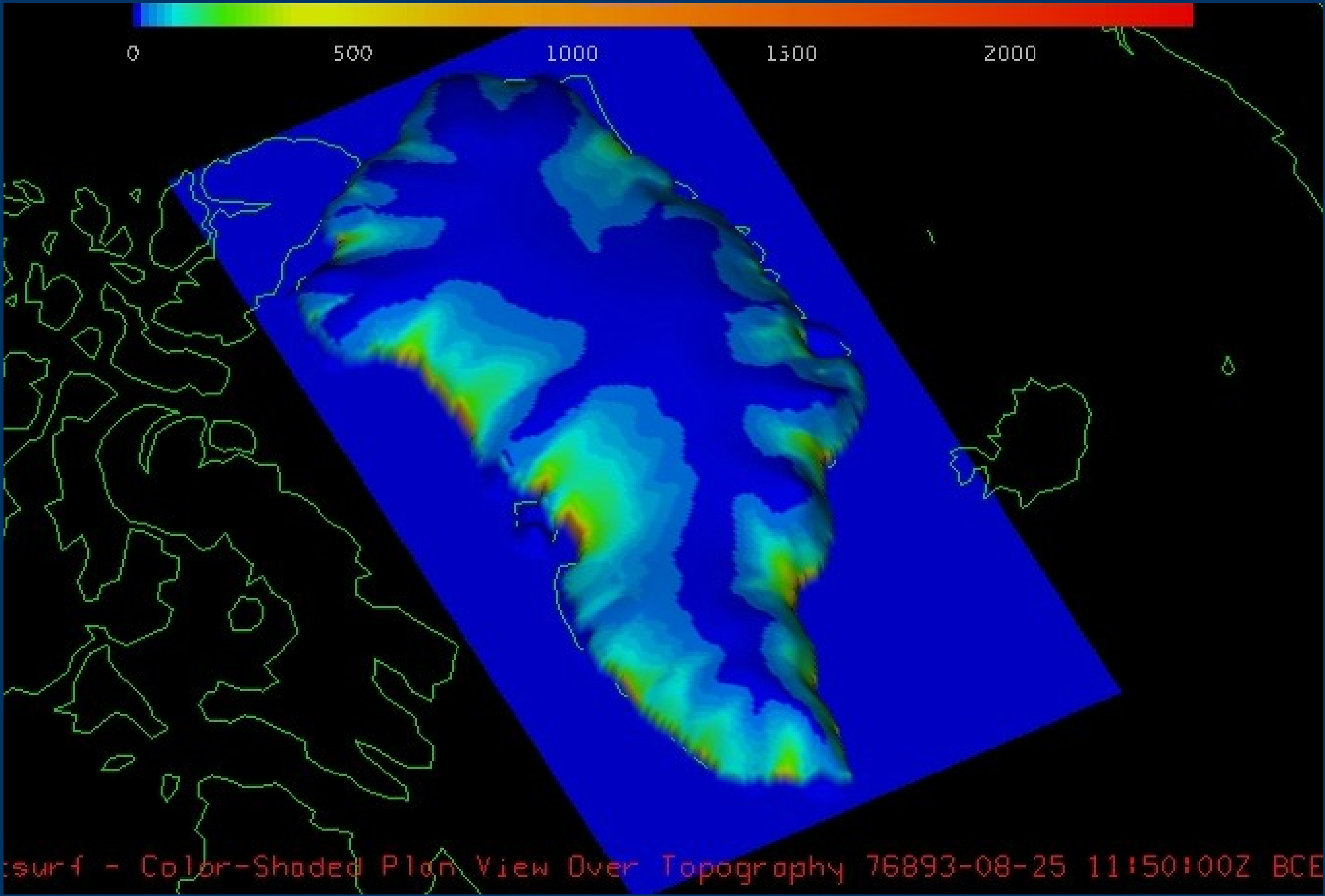
Advantages of IDV

- IDV can produce visualizations that look better and are often more useful.
  - stack multiple 2D plots on top of each other
  - 3D isosurfaces
  - 3D shape of ice (using ice thickness) colored by ice velocity
  - animations

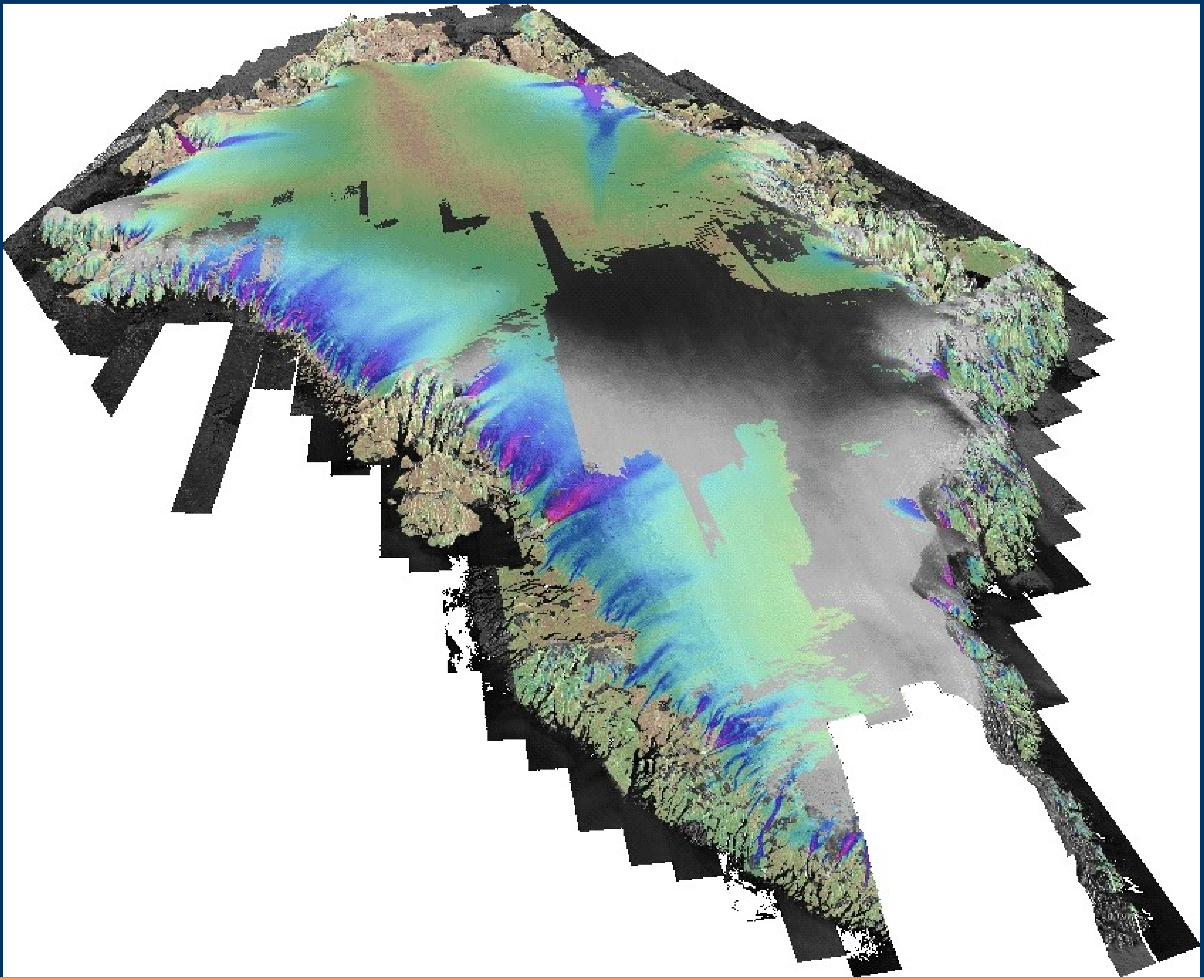
Making PISM work with IDV

- Several things necessary for me to get PISM to play nice with IDV:
  - Learn IDV (obviously).
  - Transpose *x* and *y* coordinates.
  - Split up PISM runs to save multiple NetCDF files that can be concatenated to make one big file with data over time.
    - useful for animations

Surface velocity seen with IDV



Surface velocity in the real world



Glaciers are cool but hard to study

- I took a class on field methods in glaciology.
- Getting the data that PISM uses as a given is hard.
  - measuring melt
  - mapping the terminus
  - measuring ice thickness

Thank you

- My mentor, Ed Bueler
- Patrick Webb
- Greg Newby