Basal motion beneath the Antarctic ice sheet: a comparison of linear and plastic till rheologies in a multi-modal flow model

Ed Bueler,[1] Craig Lingle,[2] Jed Brown,[2] and David Covey,[2]

[1]Department of Mathematics and Statistics, Univ. of Alaska, Fairbanks

[2]Geophysical Institute, Univ. of Alaska, Fairbanks

20 April, 2007 (EGU General Assembly 2007, Vienna)

Acknowledgements

- Thanks to David Vaughan (BAS), who generously provided our basic set of gridded data for the Antarctic ice sheet: accumulation (*Vaughan et al. 1999*), surface temperature (*Comiso et al 2000*), bed elevation (BEDMAP, *Lythe et al 2001*), thickness (based on BEDMAP and surface elevations *Liu et al 1999*), and balance velocities (*Bamber et al 2000*, based on *Budd and Warner 1996* algorithm).
- Katy Farness (OSU) and Ken Jezek (JPL) provided inSAR-derived surface velocities from MAMM.
- Martin Truffer and Christian Schoof helped with parameterizing the till yield stress under the grounded Antarctic ice sheet, but they should not be blamed for flaws in what follows.

Outline

PISM = Parallel Ice Sheet Model

an advertisement (sorry!) Authors: *Jed Brown, Ed Bueler, Craig Lingle.*

- Open Source:
  - source via Subversion at gna.org/projects/pism
  - documentation at www.dms.uaf.edu/~bueler/PISMdocinstall.htm
- Under active development. Designed primarily for Antarctic ice sheet. But feel free to try it!
- Run on up to 480 processors and on $\geq$ five different supercomputers.
- Some actual *uses* so far:
  - thermocoupled verification and EISMINT II spokes re-analysis (*Bueler, Brown, Lingle, to appear* J. Glaciol.)
  - ISMIP-HEINO (*Calov et al poster*; yesterday!)
  - Ross ice shelf model (i.e. reproduction of EISMINT I result)
  - Ongoing Antarctic ice sheet modelling . . .

Equations for multi-modal thermocoupled shallow flow: THESE APPLY EVERYWHERE

- map-plane conservation of mass
- incompressibility
- 3D (shallow approximation of) conservation of energy including bulk strain heating and friction heating from basal sliding
- 3D computation of age and grain size (latter using Vostok core relation between age and grain size; *EPICA 2004, Nature*)
- computation of basal melt water from conservation of energy; local (column-wise) conservation of melt water; freeze-on can occur
- earth deformation by method in (*Lingle and Clark 1985*); see also (*Bueler, Lingle, Brown, to appear* Ann. Glaciol.)

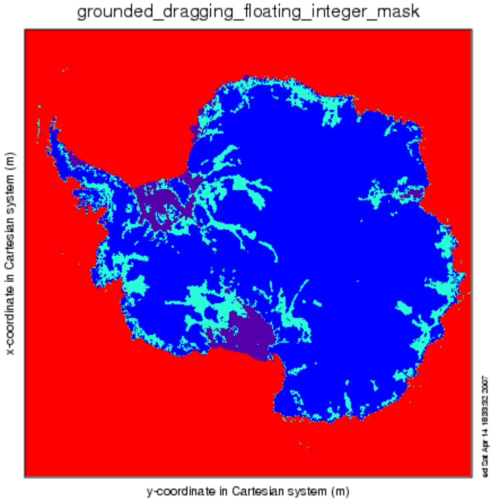
Equations for multi-modal thermocoupled shallow flow: (FLOW MODE)-DEPENDENT

(i) in interior ice sheet we
- apply shallow ice approximation to determine velocity; "SIA equations"
- use Goldsby-Kohlstedt (*2001*) flow law

(ii) in ice streams we
- apply shallow longitudinal stress balance equations with either linear basal drag (*MacAyeal 1989*) or plastic till (*Joughin,MacAyeal,Tulaczyk 2004* and *Schoof 2006*) to determine velocity nonlocally; "MMS equations"
- use a Glen flow law ($n = 3$) with Paterson-Budd temperature dependence

(iii) ice shelves are ice streams *sans* basal drag, so also MMS! (*Morland 1987*)

The which-type-of-flow mask



Shown on 14km grid; $401 \times 401 \times 241$ 3D grid.

What is *not* in PISM

- full Stokes equations (i.e. *sans* shallowness assumptions) or higher-order shallowness (e.g. *Blatter 1995* approximation)
- polythermal ice
- map-plane conservation of basal water (compare *J. Johnson PhD thesis 2002*)
- anisotropic flow laws
- Goldsby-Kohlstedt in streams and shelves [*because of a technical issue*: what is effective viscosity in G.-K.?]
- damage-mechanics-based calving criteria

The Morland-MacAyeal-Schoof (MMS) equations

Let H = thickness, h = surface elevation, $\mathbf{u} = (u, v)$ = depth-averaged horizontal velocity.

$$2\left[\nu H(2u_x + v_y)\right]_x + \left[\nu H(u_y + v_x)\right]_y + F_{b,x} - \rho g H h_x = 0$$
$$2\left[\nu H(2v_y + u_x)\right]_y + \left[\nu H(u_y + v_x)\right]_x + F_{b,y} - \rho g H h_y = 0$$

where $\mathbf{F}_b = (F_{b,x}, F_{b,y})$ is shear stress (force, to a mathematician) on base of ice from sliding:

$$\mathbf{F}_b = \begin{cases} 0 & \text{ice shelf (\textit{Morland 1987})} \\ -\beta \mathbf{u} & \text{ice stream with linearly-viscous till (\textit{MacAyeal 1989})} \\ -\tau_c \mathbf{u}/|\mathbf{u}| & \text{ice stream with plastic till (\textit{Schoof 2006})} \end{cases}$$

Note (*Schoof 2006*)[1] plastic till is different

For plastic till, determination of the location of sliding is simultaneous with the computation of the sliding velocity.

I.e. Schoof formulated the *free boundary problem* for ice streams by identifying this functional

$$J[\mathbf{v}] = \int_\Omega \frac{2BH}{p} \left[\dot\epsilon_{ij}\dot\epsilon_{ij}/2 + \dot\epsilon_{ii}^2/2\right]^{p/2} + \tau_c|\mathbf{v}| + \rho g H \nabla h \cdot \mathbf{v} - \int_{\partial\Omega} \mathbf{F}_{\text{cf}} \cdot \mathbf{v}$$

where $p = 1 + 1/n$, $\dot\epsilon_{ij}$ is the strain rate tensor, Ω is the whole ice-covered region, and $\mathbf{F}_{\text{cf}}$ is the force on the edge of the ice (i.e. at calving front). $J[\cdot]$ is *minimized by the actual velocity among all possible depth-averaged velocity fields* over Ω.

Thus one *does not* need a type-of-flow mask. One *does* need to model τ_c. For ice shelves $\tau_c = 0$.

[1] J. Fluid Mech. "A variational approach to ice stream flow"

Verification for ice sheets

DEFINITION:

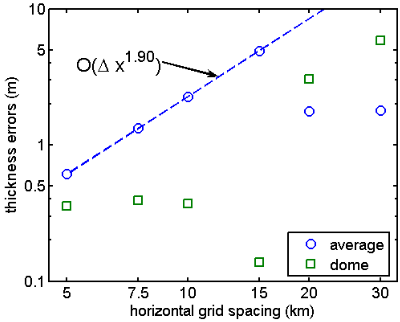
Verification is measuring the difference between numerical results and exact solutions and measuring the rate at which numerics converge to exact continuum values as grid is refined.

Are there enough exact solutions? Perhaps so! These are built into PISM:

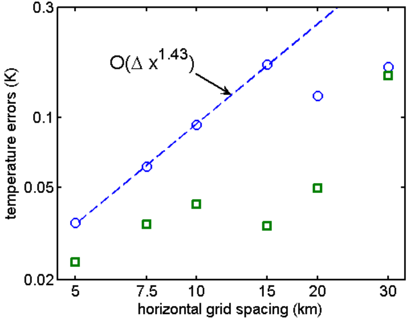
- similarity exact solutions to the isothermal SIA (*Halfar 1983*, *Bueler et al 2005*)
- "manufactured" exact solutions to the isothermal *and thermocoupled* SIA (*Bueler et al 2005*, *Bueler et al, to appear* J. Glaciol.)
- "manufactured" exact solution to MMS with linear drag (*Brown MS Thesis 2006*)
- exact solution to MMS with plastic till (*Schoof 2006*)

Convergence under grid refinement: an SIA example

Verification of PISM's approximation of thermocoupled SIA is done in (*Bueler et al, to appear* J. Glaciol.).



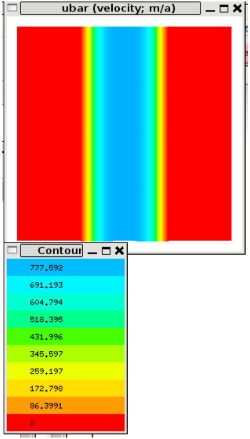
thickness errors



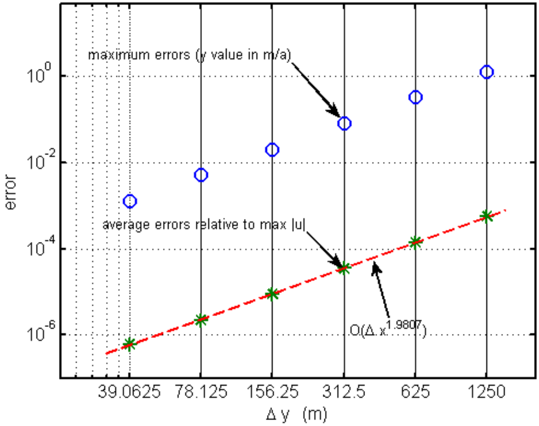
temperature errors

PISM can approximate plastic till ice streams

Compare to exact solution in (*Schoof 2006*) of plastic till MMS.



velocity



velocity errors

Validation

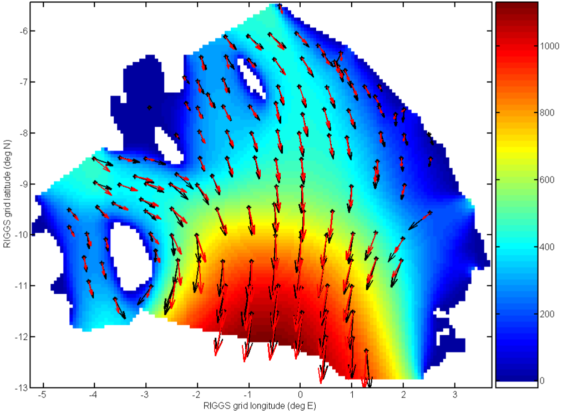
DEFINITION:

Validation is measuring the difference between numerical model results and real observations in circumstances where the real system is well-observed

Ice shelf flow: validation

Recall ice shelf part of EISMINT I (*MacAyeal et al 1996*). Color shows PISM's modeled speed (m/a) on Ross Ice Shelf with 6.8 km grid.

- **black** arrows are observed velocities (RIGGS)
- **red** arrows are PISM-modeled velocities at same points



6.8 km seems fine enough to resolve (ice stream/glacier) inputs to shelf

Initialization

By "initialization" of a time-dependent Antarctic flow model, we really mean the creation of a model of the current state of the Antarctic ice sheet. *This our current goal for PISM.*

- initializing means *solving obligatory inverse problems*
- we must "fill in" the following to initialize:
  - (i) temperature (note long "spin-up" to meet advection time scale even at steady state)
  - (ii) basal condition (i.e. drag coefficient or yield stress)
  - (iii) age and grain size (needed by G.-K. flow law)
  - (iv) melt/freeze rates under ice shelves
  - (v) distribution of basal water under grounded sheet
- *then* flow equations determine velocities . . .

Determining basal conditions in an ice sheet model

Recall MMS equations. If thickness H, surface elevation h, viscosity ν, and depth-averaged velocities $\mathbf{u} = (u, v)$ are known, then MMS equations determine basal stress $\mathbf{F}_b$.

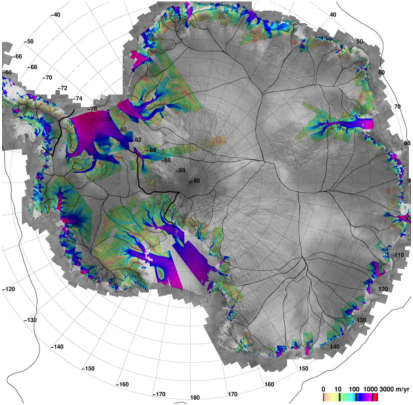
PISM model state—other ice sheet models, too—includes H, h, and an estimate of ν (from flow law and temperature).

Whence depth-averaged $\mathbf{u}$ for current state of an ice sheet?

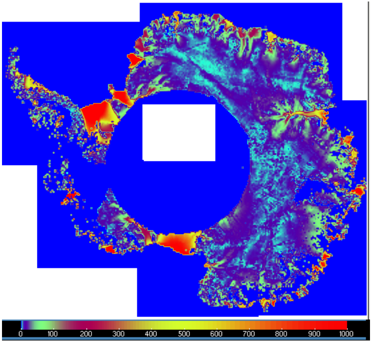
1. From surface velocities $\mathbf{u}(z = h)$: Subtract SIA-computed total shear deformation to determine basal sliding. Then compute depth-averaged $\mathbf{u}$.
2. From balance velocities: Use these for $\mathbf{u}$. (Need to trust observed accumulations and assume steady state for sheet.)

Observed surface velocity

Coverage: is it good enough to determine basal stress for all grounded points yet?



interferometry and speckle-tracking (*Rignot et al 2004*)

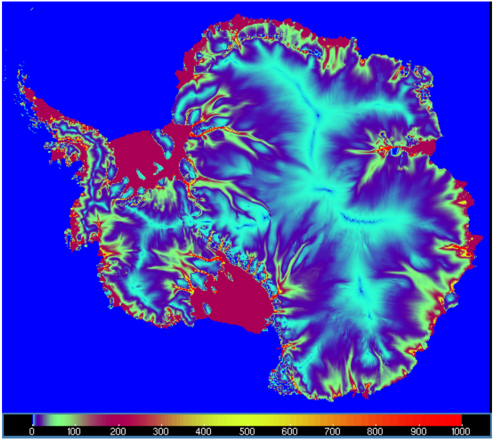


Modified Antarctic Mapping Mission (SAR interferometry) (*K. Farness and K. Jezek*)

Balance velocities

Stuck with these for short term. *Flaws*:

- assume steady state
- depend on measured accumulation rates
- assume ice flows downhill

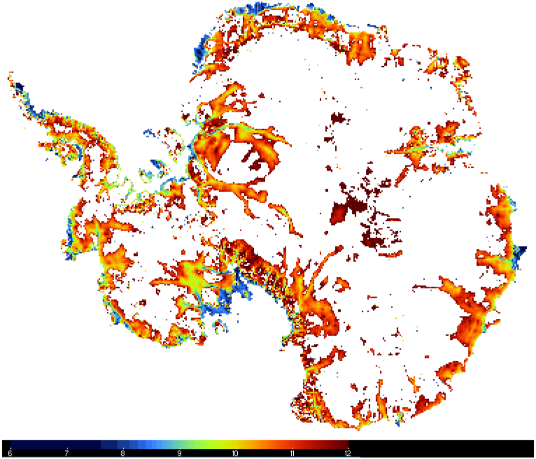


Drag coefficient from balance velocities

Given balance velocities, compute $\mathbf{F}_b$ as above.

Assume linearly-viscous till. MMS eqns give β_x, β_y so that $\mathbf{F}_b = (-\beta_x u, -\beta_y v)$. Compute weighted average $\beta = (u^2\beta_x + v^2\beta_y)/|\mathbf{u}|^2$.

Figure shows $\log_{10}(\beta)$ where β in Pa s m^{-1} on 14km model grid; β was removed if outside of $[10^5, 10^{13}]$. Compare $\beta = 2.0 \times 10^9$ used by Hulbe & MacAyeal.



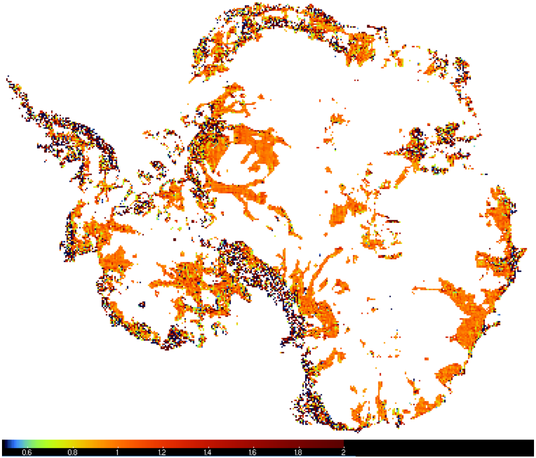
Ratio of drag components

Note β_x, β_y can come out different. So

$$\frac{\beta_y}{\beta_x} \approx 1$$

is a standard for consistency.

Figure shows β_y/β_x.



What if no $\mathbf{u}$ available?

As noted, an alternative approach uses MMS to compute basal sliding velocity using Schoof's plastic till free boundary problem.

PISM's model state includes H, h, and estimate of ν, as needed. To estimate yield stress, we recompute

$$\tau_c = c_0 + \mu(\rho g H - \lambda p_w)$$

where c_0 is cohesion ($= 0.2$ bar) and $\mu = \tan(25^\circ)$ is "friction angle". Here $\lambda = 0$ when base is frozen and λ increases to one linearly as a function of computed effective basal melt water thickness (stored in till). We estimate pore water pressure p_w in till this way:

$$p_w = (0.85 + 0.1 \min\{\max\{-b, 0\}/1000, 1\})\rho g H$$

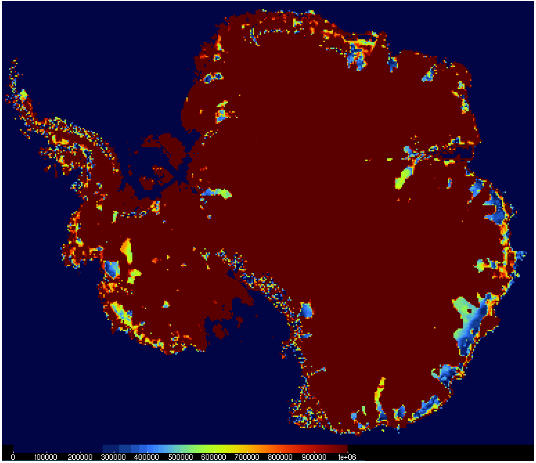
where ρ_{sw} is density of sea water and b is bed elevation (negative below sea level).

We compute map of yield stress τ_c at the start of every temperature time step.

Plastic till case: modeled yield stress

It works and is stable. Figure shows τ_c in Pa.

Difficulty: No sliding without warmth at base. No warmth at base without the frictional heat from sliding.

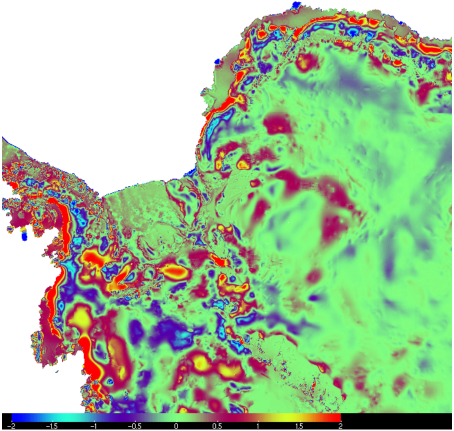


Conclusions

- Based on verification and validation, PISM is doing an adequate job of solving the MMS equations in streams and shelves (and the SIA eqns for rest of sheet).
- Given depth-averaged horizontal velocities $\mathbf{u} = (u, v)$ the MMS equations determine the basal shear stress; either linearly-viscous ("βu") or plastic ("$\tau_c u/|\mathbf{u}|$") till. Determining β directly this way seems surprisingly well-behaved.
- Existing source for $\mathbf{u} = (u, v)$ is balance velocities. Future source: observed surface velocity.
- Schoof's plastic till MMS equations determine both *where* there is sliding and *how fast*; "mask" notion is no longer needed for grounded ice. But one must model the till yield stress itself.
- With upwinding of the mass conservation equation in the MMS regions, we observe no stability problems whatsoever. We use adaptive time-stepping based on CFL on all transport and diffusivity in SIA regions.

dH/dt over a 100 year run on a 5km grid

- 5km grid of whole Antarctic sheet
- 480 processors at ARSC
- run for 2.5 hours
- 100 model years
- time steps = 4 model days
- thermocoupled SIA using GK flow law; no MMS
- 3D grid for temperature is $1121 \times 1121 \times 241$
- 500 million unknowns
- MMS equations parallelize, but not as well



Questions

- Is it correct *to low order* to superpose the velocity field from the SIA equations with the MMS velocities?
- What is the best Krylov scheme for the MMS equations? The best preconditioner?
- How accurate is the accumulation map for Antarctica, from which one computes balance velocities?
- Could someone produce some "surface velocity-guided" balance velocities for the whole Antarctic ice sheet, (especially) as a tool to assess the quality of the accumulation map?

THANKS FOR YOUR ATTENTION OR TOLERANCE!

Information about PISM = Parallel Ice Sheet Model

- website
  `gna.org/projects/pism`
- source via Subversion by
  `svn co http://svn.gna.org/svn/pism/trunk pism`
- documentation at
  `www.dms.uaf.edu/~bueler/PISMdocinstall.htm`