Multi-Modal Flow in a Thermocoupled Model of the Antarctic Ice Sheet, with Verification

Craig Lingle[1] Jed Brown[2] Ed Bueler[2]

[1]Geophysical Institute
University of Alaska Fairbanks, USA

[2]Department of Mathematics and Statistics
University of Alaska Fairbanks, USA

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(note: MINOR ADDITIONS TO TALK AS GIVEN)

Outline

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1. Continuum model for multi-modal thermocoupled flow
   - Constitutive relations and evolution equations
   - Shallow ice approximation (SIA) inland flow
   - Ice shelf and ice stream flow
2. Verification
3. Inputs to the model (for Antarctica)
4. Some preliminary results for current state of Antarctica
   - Initialization
   - Velocity and temperature fields
   - Estimation of basal drag

Goldsby-Kohlstedt (2001) constitutive relation

Used in the interior of the ice sheet

Four flow regimes

Each term is like Arrhenius-Glen-Nye flow law, but with different stress exponent. Note $\dot\epsilon$ is 2nd invariant of strain rate tensor $\dot\epsilon_{ij}$.

$\dot\epsilon_{\text{diff}}$	diffusion creep $(n = 1)$	grain size dependent
$\dot\epsilon_{\text{gbs}}$	grain-boundary sliding $(n = 1.8)$	grain size dependent
$\dot\epsilon_{\text{basal}}$	basal glide $(n = 2.4)$	
$\dot\epsilon_{\text{disl}}$	dislocation climb $(n = 4)$	

A nontrivial combination

$$\dot\epsilon = \dot\epsilon_{\text{diff}} + \left(\frac{1}{\dot\epsilon_{\text{gbs}}} + \frac{1}{\dot\epsilon_{\text{basal}}}\right)^{-1} + \dot\epsilon_{\text{disl}}$$

Glen's flow law

1. used for ice stream/shelf flow
2. used for verification
   - time dependent exact solutions to thermocoupled SIA
   - time independent exact solutions for ice streams

Arrhenius-Glen-Nye form

$$\dot{\epsilon}_{ij} = A(T^*)\sigma^{n-1}\sigma_{ij}$$

$A(T^*)$	softness factor	σ_{ij}	stress deviator tensor
T^*	homologous temperature	σ	second invariant of σ_{ij}
n	stress exponent		

we use Paterson and Budd (1982) form for $A(T^*)$

Inverse Glen's flow law needed for shelf/stream flow

Stress in terms of strain rate

$$\sigma_{ij} = 2\nu(\dot{\epsilon}, T^*)\dot{\epsilon}_{ij}$$

Effective viscosity

For Glen's flow law,

$$\nu(\dot{\epsilon}, T^*) = \frac{1}{2} A(T^*)^{-1/n} \dot{\epsilon}^{\frac{n-1}{n}}$$

Note

It is difficult to invert the Goldsby-Kohlstedt flow law.

Mass-balance and conserv. of energy *solved everywhere*

Map-plane mass-balance equation

$$\frac{\partial H}{\partial t} = M - \nabla \cdot \mathbf{Q} \quad \text{where } \mathbf{Q} = \overline{\mathbf{U}}\, H$$

H	thickness	$\mathbf{Q}$	map-plane hor. flux
M	ice-equiv. accum. rate	$\overline{\mathbf{U}}$	vert.-averaged hor. vel.

Conservation of energy (temperature) equation

$$\frac{\partial T}{\partial t} + \mathbf{U} \cdot \nabla T + w \frac{\partial T}{\partial z} = K \frac{\partial^2 T}{\partial z^2} + \text{(strain-heating)}$$

T	ice temperature	$\mathbf{U}$	horizontal velocity
K	conductivity of ice	w	vertical velocity

Velocity determined locally for inland (SIA) ice sheet

Get velocity in SIA by vertically-integrating this:

$$\frac{\partial \mathbf{U}}{\partial z} = -2F(\sigma, T^*, \dots)P\nabla h$$

$\sigma = \rho g(h-z)\lvert\nabla h\rvert$	shear stress	T^*	homol. temperature
$P = \rho g(h-z)$	pressure	h	surface elevation

(*Note*: Add basal velocity $\mathbf{U}_b$, too!)

Note: all isotropic flow laws have form

$$\dot{\epsilon}_{ij} = F(\sigma, T^*, \dots)\sigma_{ij}$$

where "..." might include grain size, pressure, etc.

Velocity determined "globally" in streams and shelves

MacAyeal-Morland equations for Glen law

Velocity in ice shelves and streams is depth-independent. Solve a boundary-value problem at each time:

$$\left[2\nu H(2u_x + v_y)\right]_x + \left[\nu H(u_y + v_x)\right]_y - \beta u = \rho g H h_x$$
$$\left[2\nu H(2v_y + u_x)\right]_y + \left[\nu H(u_y + v_x)\right]_x - \beta v = \rho g H h_y$$

where effective viscosity *depends on velocity and temperature*:

$$\nu = \frac{\overline{B}}{2}\left[\frac{1}{2}u_x^2 + \frac{1}{2}v_y^2 + \frac{1}{2}(u_x + v_y)^2 + \frac{1}{4}(u_y + v_x)^2\right]^{\frac{1-n}{2n}},$$

$$\overline{B} = \left(\text{vertical average of } A(T^*)^{-1/n}\right)$$

Notes on basal motion: linear (for now)

Thermally-activated

If the bed temp is below pressure-melting then no sliding.

Inland ice sheet flow

Assume till has viscosity ν and thickness L. Basal velocity from basal effective shear stress:

$$\text{(basal velocity)} = \frac{L}{\nu}\text{(basal stress)}$$

Ice stream flow

Basal stress determined by friction parameter β ($\beta = 0$ for shelves):

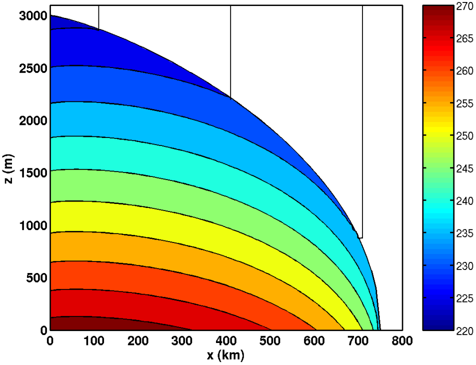
$$\text{(basal stress)} = \beta\text{(basal velocity)}$$

Outline

Verification of the SIA numerics

Time-dependent exact solution to thermocoupled SIA equations

H, T chosen $\longrightarrow$ [compute accumulation, velocity, strain-heating, etc. which satisfy all eqns]



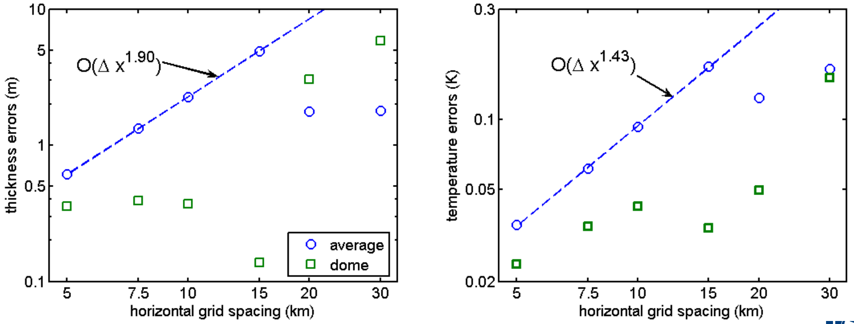
Reference

Bueler, Kallen-Brown, Lingle, *Exact solutions to thermocoupled ice-sheet models . . .*, submitted soon!

Convergence under grid refinement

Because we know exact solution,

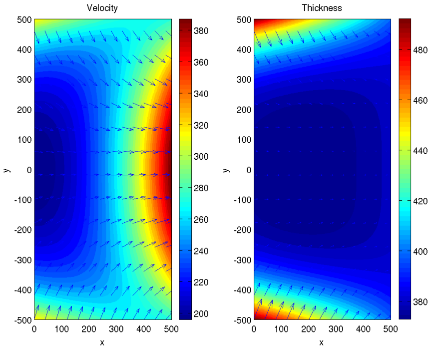
- numerical errors (thickness and temperature) are known and
- convergence rate under grid refinement can be measured.

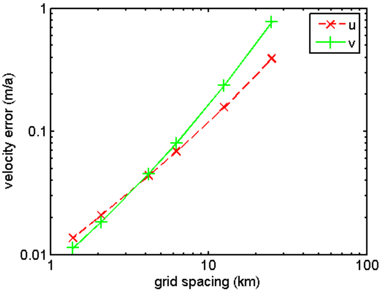


Verification of the (dragging) ice shelf numerics

Exact solution to the MacAyeal-Morland equations

u, v, H chosen $\longrightarrow$ (compute drag which satisfies eqns)

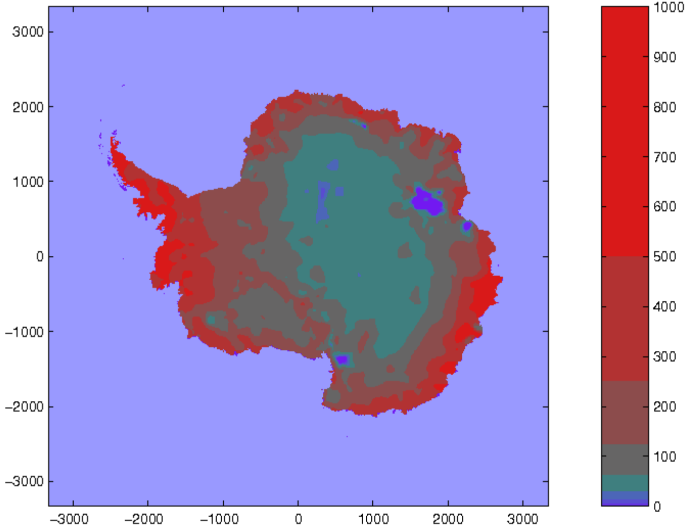




Outline

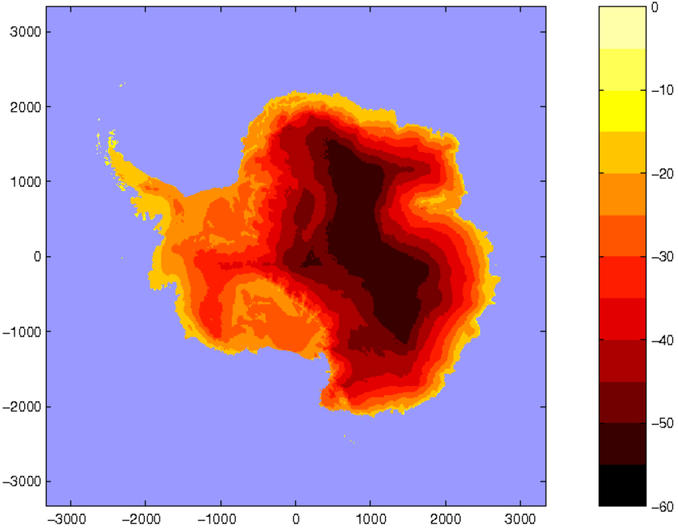
Accumulation (m/a)

Vaughan et al., 1999, provided by British Antarctic Survey



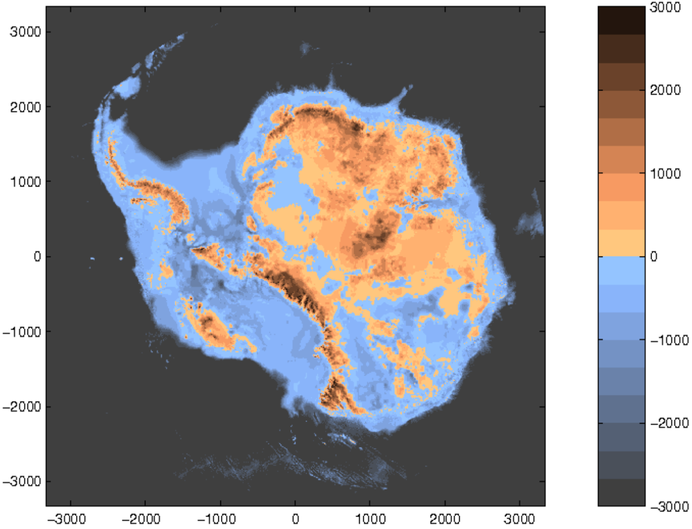
Surface temperature (K)

Comiso et al (2000), provided by BAS



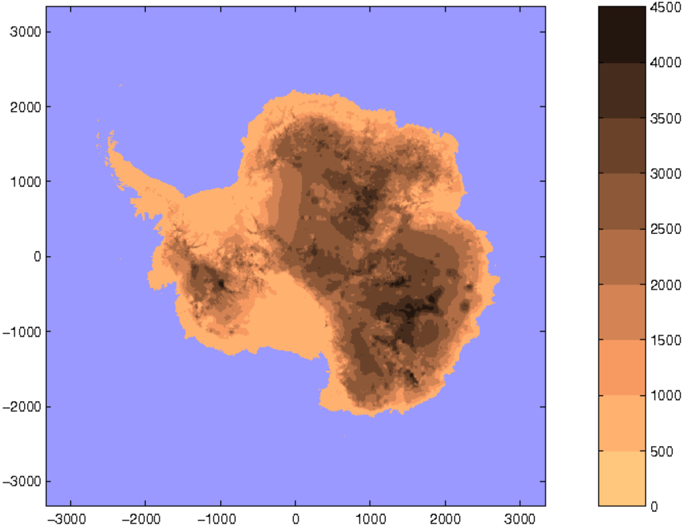
Bed elevation (m)

BEDMAP, Lythe et al (2001), provided by BAS



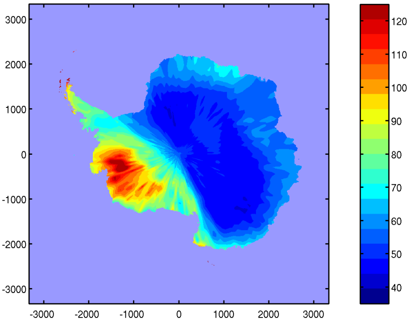
Thickness (m)

Based on BEDMAP and surface elevations from Liu et al (1999), and provided by BAS

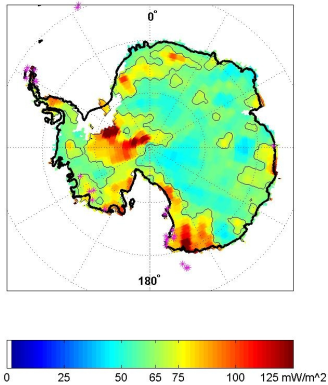


Geothermal flux (mW/m^2)

Shapiro & Ritzwoller (2004; Earth Planetary Sci. Let.); results computed from this one:

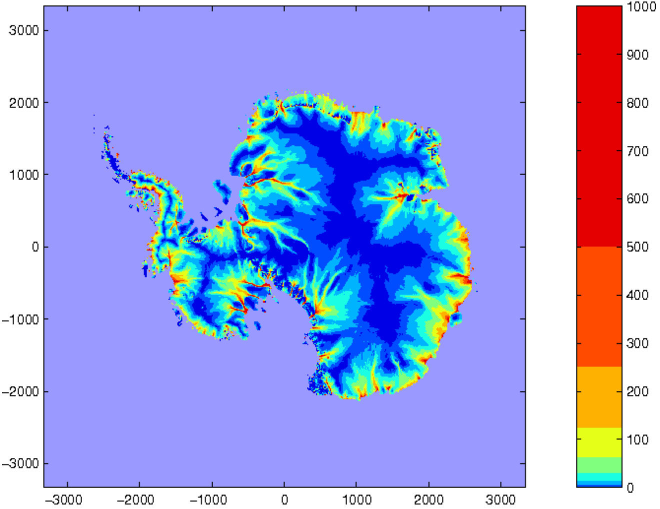


Fox Maule et al. (2005; Science):



Balance velocity is used for flow mode "mask"

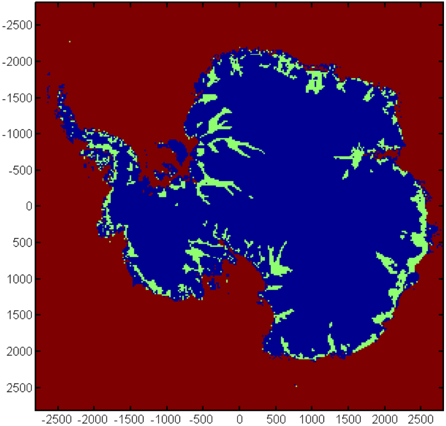
Bamber et al (2000) based on Budd and Warner (1996) algorithm



The which-type-of-flow mask

Flow type is determined for current state by (sliding velocity) = (balance velocity) − (Goldsby-Kohlstedt deformational velocity):

- **red** if ice is floating (or ice-free ocean)
- **blue** (inland SIA) if (sliding) $\leq 40\,\mathrm{m\,a^{-1}}$
- **green** (ice stream) if (sliding) $> 40\,\mathrm{m\,a^{-1}}$



Outline

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Initializing a numerical ice sheet model for real ice sheets *means solving obligatory inverse problems*

Boundary data available to modellers:

surface elevation, thickness, bed elevation, accumulation, surface temperature, geothermal flux [*from other models*], mass-balance velocities [*assumptions plus computations*], . . .

***SPARSE* data at depth (e.g. ice core data)**

temperature, age, grain size, basal condition [*very sparse*], . . .

Initializing *means solving inverse problems* cont.

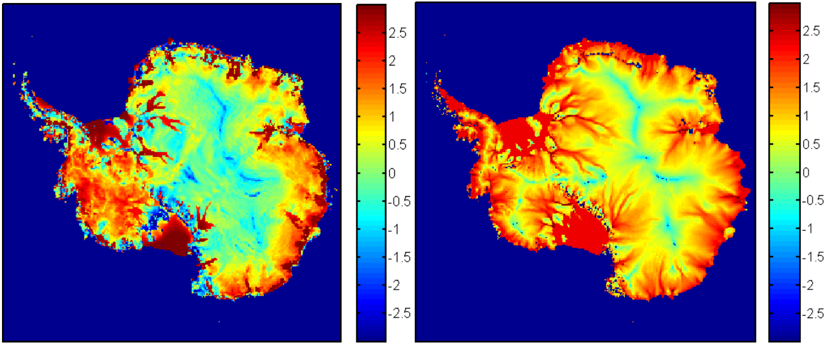
Must "fill in" (or guess) to initialize simulation

- temperature (long "spin-up" to meet advection time scale)
- basal condition (drag)
- age and grain size (needed by G.-K. flow law)

Reminder: With above fields, flow equations determine velocity field, but velocity field effects temperature and basal conditions. . .

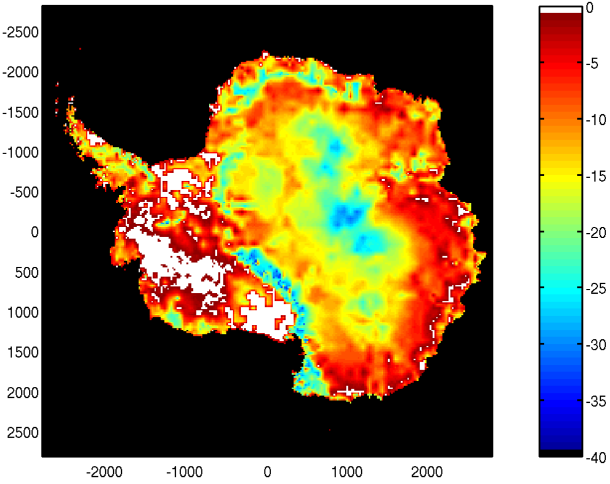
Modeled horizontal velocity [*preliminary*]

- left: $\log_{10}$ of vertically-averaged horizontal speed
- right: $\log_{10}$ of mass-balance speed (Bamber et al 2000)



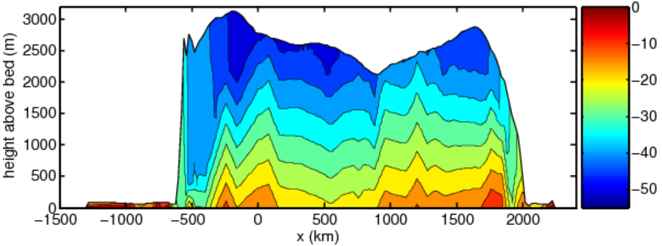
Modeled basal homologous temperature [*preliminary*]

(degrees C below pressure melting)

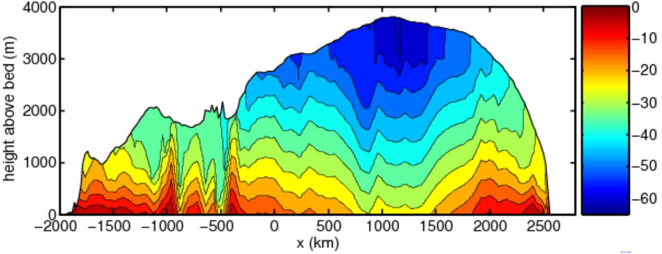


Modeled temp along sections through S pole [*preliminary*]

Along $0°$–$180°$:

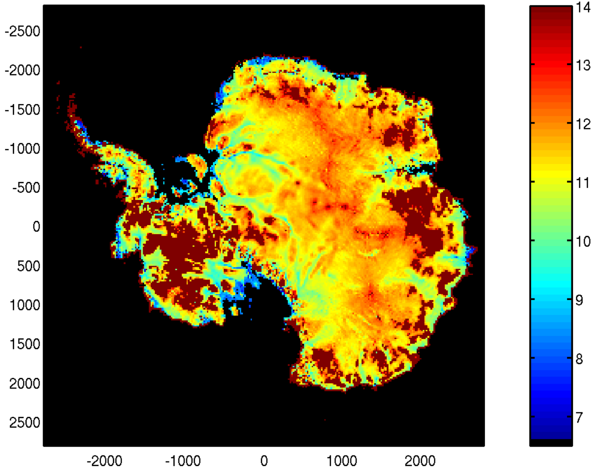


Along $90°$W–$90°$E:



Basal drag assuming linear law: (stress) $= -\beta \mathbf{U}_{\text{sliding}}$

$\log_{10}(\beta)$ where β is in units $\mathrm{Pa\,s\,m^{-1}}$. Compare constant value $2.0 \times 10^9\ \mathrm{Pa\,s\,m^{-1}}$ in (Hulbe and MacAyeal 1998). *Preliminary.*



How the last slide was created

Getting basal drag from balance velocities and the SIA

1. deformational (SIA) velocities are computed at all grounded points (using Goldsby-Kohlstedt)
2. average deformational velocity is subtracted from mass-balance velocity to give a sliding velocity
3. this sliding velocity is put into the MacAyeal-Morland equations at all grounded points to determine the drag coefficient which would give this much sliding

Notes

- If deformational velocities exceed balance velocities then get negative drag! Here we set $\beta = 10^{14}\,\mathrm{Pa\,s}\,m^{-1}$ in that case.
- Effect of high geothermal flux in Amundsen sector (from Shapiro and Ritzwoller map) is clear.

Summary

Our model

1. is multi-modal (SIA and MacAyeal-Morland eqns for flow)
2. is verifiable (and verified) for each mode of flow
3. solves all equations in parallel (PETSc)
4. allows choice of grid resolution at run time
5. includes new earth deformation model (*that's another talk...*)

Planned directions

1. consequences of different geothermal flux maps
2. improved basal dynamics
3. depth-dependent density, calving criteria, ...
4. moving boundary between SIA and ice stream flow