The challenge of prediction: developing a polar ice sheet model capable of simulating the responses of ice shelves and ice streams to a warming ocean

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Outline

I. Our model is called PISM (Parallel Ice Sheet Model). It's licensed as public domain code! You can try it yourself!

II. How ice sheets deform in response to stress: The constitutive equation or flow law.

III. Ice sheets evolve through time in accordance with the continuity eq'n, which conserves mass, and the temperature eq'n, which conserves energy.

IV. The big challenge: simulating inland ice-sheet flow, ice Ice-stream flow, and ice-shelf flow, all in the same model.

V. Input data, and a preliminary demonstration run.

Goldsby-Kohlstedt (2001) constitutive relation

Used in the interior of the ice sheet

Four flow regimes

Each term is like Arrhenius-Glen-Nye flow law, but with different stress exponent. Note $\dot\epsilon$ is 2nd invariant of strain rate tensor $\dot\epsilon_{ij}$.

$\dot\epsilon_{\text{diff}}$	diffusion creep $(n = 1)$	grain size dependent
$\dot\epsilon_{\text{gbs}}$	grain-boundary sliding $(n = 1.8)$	grain size dependent
$\dot\epsilon_{\text{basal}}$	basal glide $(n = 2.4)$	
$\dot\epsilon_{\text{disl}}$	dislocation climb $(n = 4)$	

A nontrivial combination

$$\dot\epsilon = \dot\epsilon_{\text{diff}} + \left(\frac{1}{\dot\epsilon_{\text{gbs}}} + \frac{1}{\dot\epsilon_{\text{basal}}}\right)^{-1} + \dot\epsilon_{\text{disl}}$$

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Glen's flow law

1. used for ice stream/shelf flow
2. used for verification
   - time dependent exact solutions to thermocoupled SIA
   - time independent exact solutions for ice streams

Arrhenius-Glen-Nye form

$$\dot{\epsilon}_{ij} = A(T^*)\sigma^{n-1}\sigma_{ij}$$

$A(T^*)$	softness factor	σ_{ij}	stress deviator tensor
T^*	homologous temperature	σ	second invariant of σ_{ij}
n	stress exponent		

we use Paterson and Budd (1982) form for $A(T^*)$

Inverse Glen's flow law needed for shelf/stream flow

Stress in terms of strain rate

$$\sigma_{ij} = 2\nu(\dot{\epsilon}, T^*)\dot{\epsilon}_{ij}$$

Effective viscosity

For Glen's flow law,

$$\nu(\dot{\epsilon}, T^*) = \frac{1}{2} A(T^*)^{-1/n} \dot{\epsilon}^{\frac{n-1}{n}}$$

Note

It is difficult to invert the Goldsby-Kohlstedt flow law.

Mass-balance and conserv. of energy *solved everywhere*

Map-plane mass-balance equation

$$\frac{\partial H}{\partial t} = M - \nabla \cdot \mathbf{Q} \quad \text{where } \mathbf{Q} = \overline{\mathbf{U}}\, H$$

H	thickness	$\mathbf{Q}$	map-plane hor. flux
M	ice-equiv. accum. rate	$\overline{\mathbf{U}}$	vert.-averaged hor. vel.

Conservation of energy (temperature) equation

$$\frac{\partial T}{\partial t} + \mathbf{U} \cdot \nabla T + w \frac{\partial T}{\partial z} = K \frac{\partial^2 T}{\partial z^2} + (\text{strain-heating})$$

T	ice temperature	$\mathbf{U}$	horizontal velocity
K	conductivity of ice	w	vertical velocity

Velocity determined locally for inland (SIA) ice sheet

Get velocity in SIA by vertically-integrating this:

$$\frac{\partial \mathbf{U}}{\partial z} = -2F(\sigma, T^*, \dots)P\nabla h$$

$\sigma = \rho g(h-z)\lvert\nabla h\rvert$	shear stress	T^*	homol. temperature
$P = \rho g(h-z)$	pressure	h	surface elevation

(*Note*: Add basal velocity $\mathbf{U}_b$, too!)

Note: all isotropic flow laws have form

$$\dot\epsilon_{ij} = F(\sigma, T^*, \dots)\sigma_{ij}$$

where "..." might include grain size, pressure, etc.

Velocity determined "globally" in streams and shelves

MacAyeal-Morland equations for Glen law

Velocity in ice shelves and streams is depth-independent. Solve a boundary-value problem at each time:

$$\left[2\nu H(2u_x + v_y)\right]_x + \left[\nu H(u_y + v_x)\right]_y - \beta u = \rho g H h_x$$
$$\left[2\nu H(2v_y + u_x)\right]_y + \left[\nu H(u_y + v_x)\right]_x - \beta v = \rho g H h_y$$

where effective viscosity *depends on velocity and temperature*:

$$\nu = \frac{\overline{B}}{2}\left[\frac{1}{2}u_x^2 + \frac{1}{2}v_y^2 + \frac{1}{2}(u_x + v_y)^2 + \frac{1}{4}(u_y + v_x)^2\right]^{\frac{1-n}{2n}},$$

$$\overline{B} = \left(\text{vertical average of } A(T^*)^{-1/n}\right)$$

Notes on basal motion: linear (for now)

Thermally-activated

If the bed temp is below pressure-melting then no sliding.

Inland ice sheet flow

Assume till has viscosity ν and thickness L. Basal velocity from basal effective shear stress:

$$(\text{basal velocity}) = \frac{L}{\nu}(\text{basal stress})$$

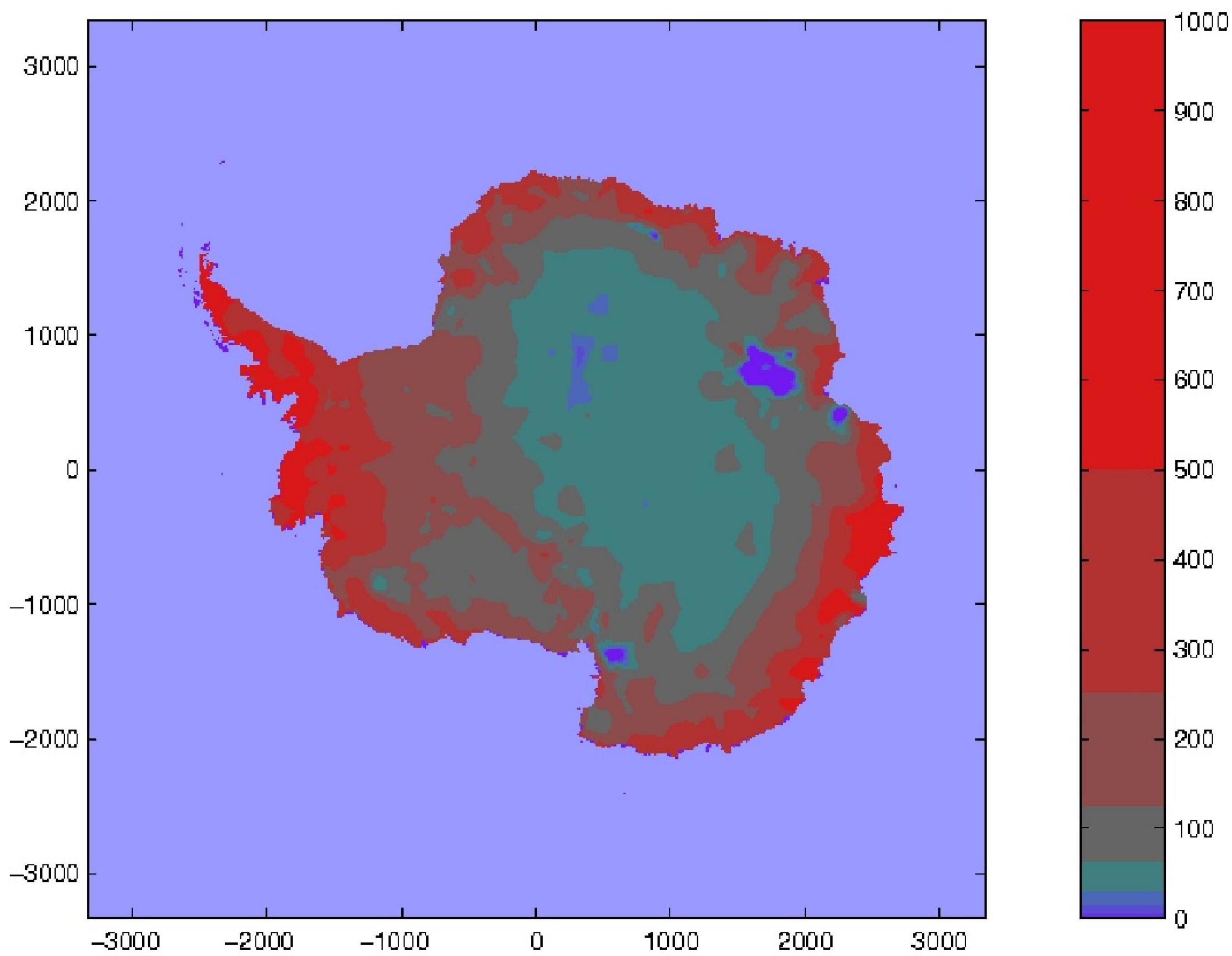
Ice stream flow

Basal stress determined by friction parameter β ($\beta = 0$ for shelves):

$$(\text{basal stress}) = \beta(\text{basal velocity})$$

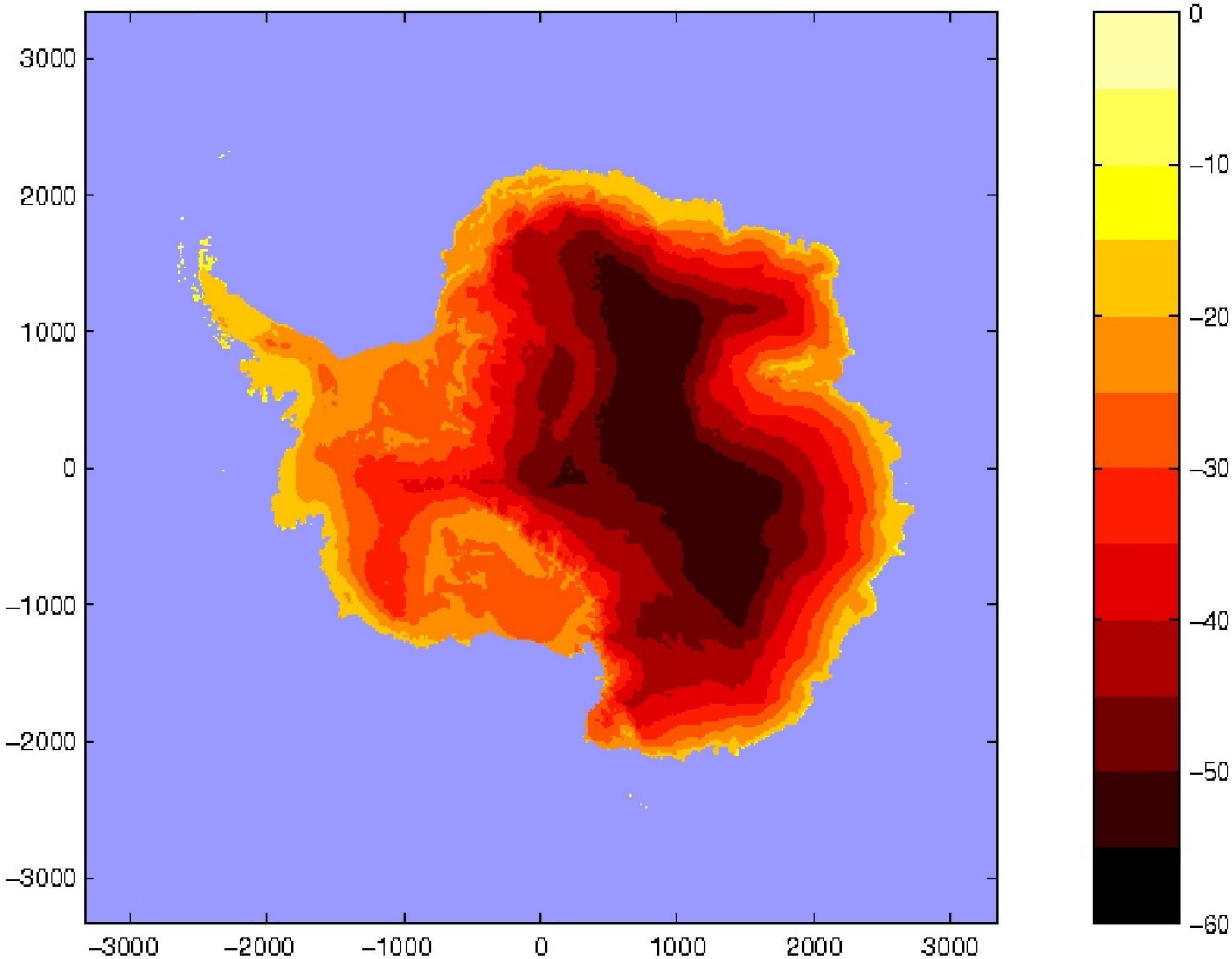
Accumulation (m/a)

British Antarctic Survey 2004



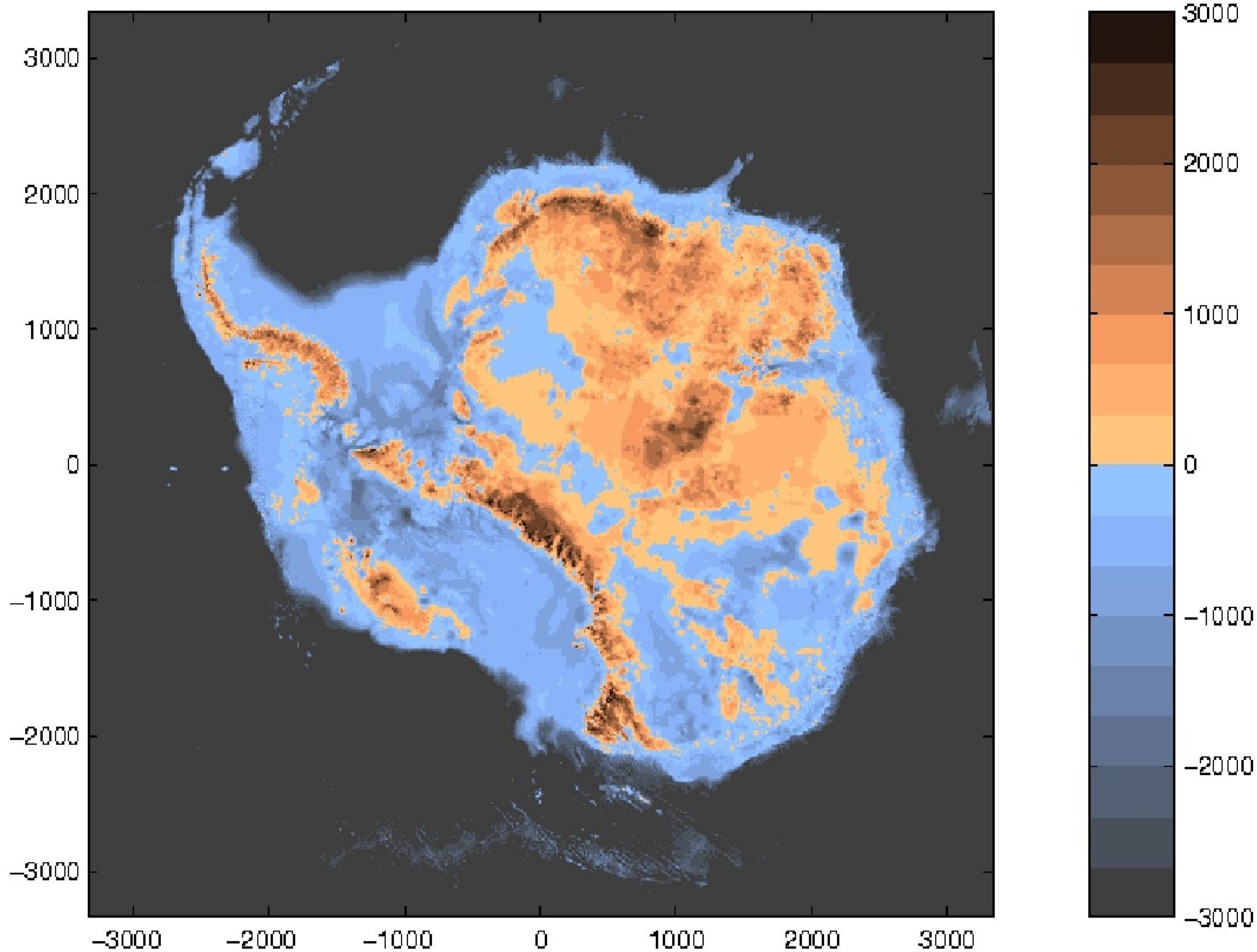
Surface temperature (K)

British Antarctic Survey 2004



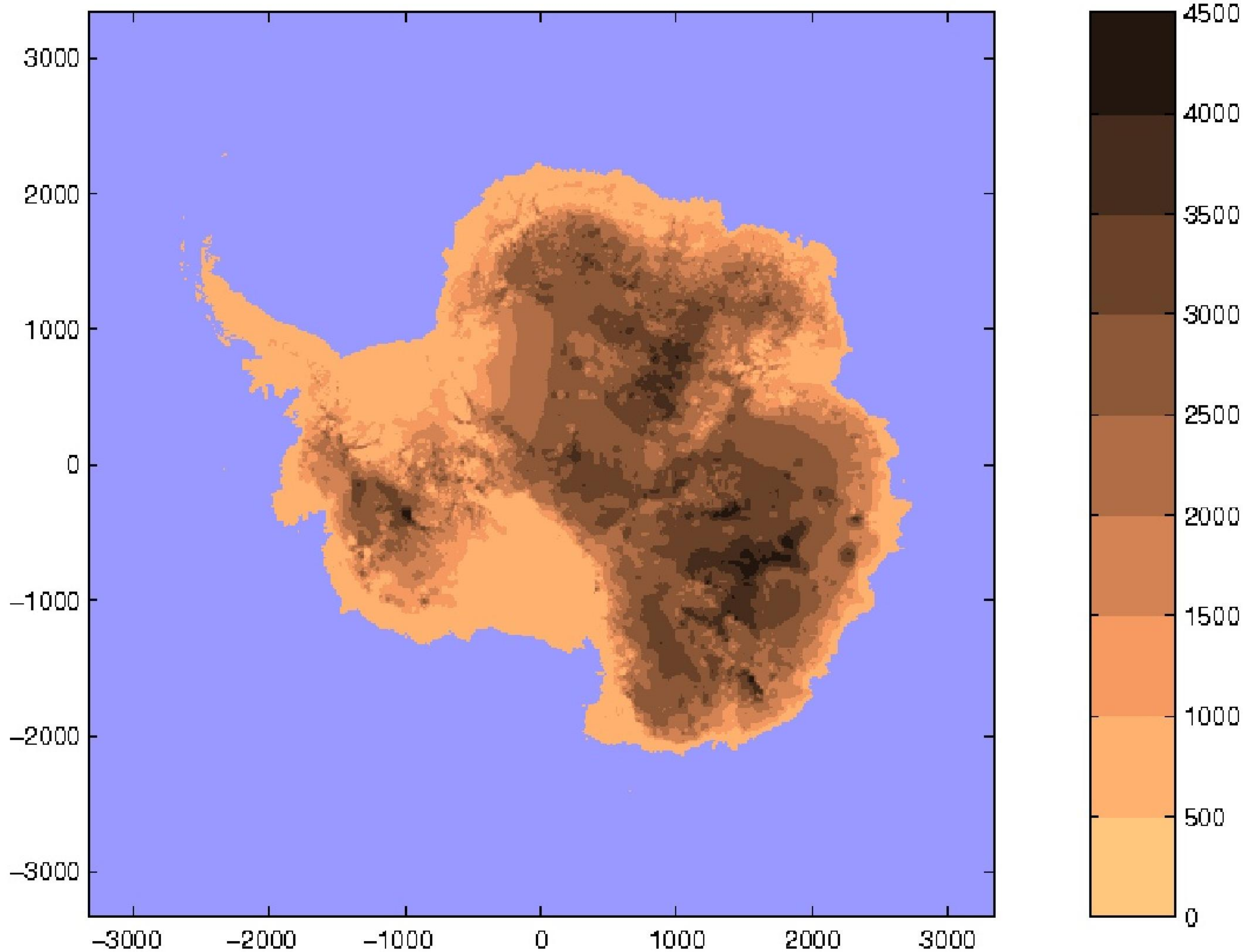
Bed elevation (m)

British Antarctic Survey 2004



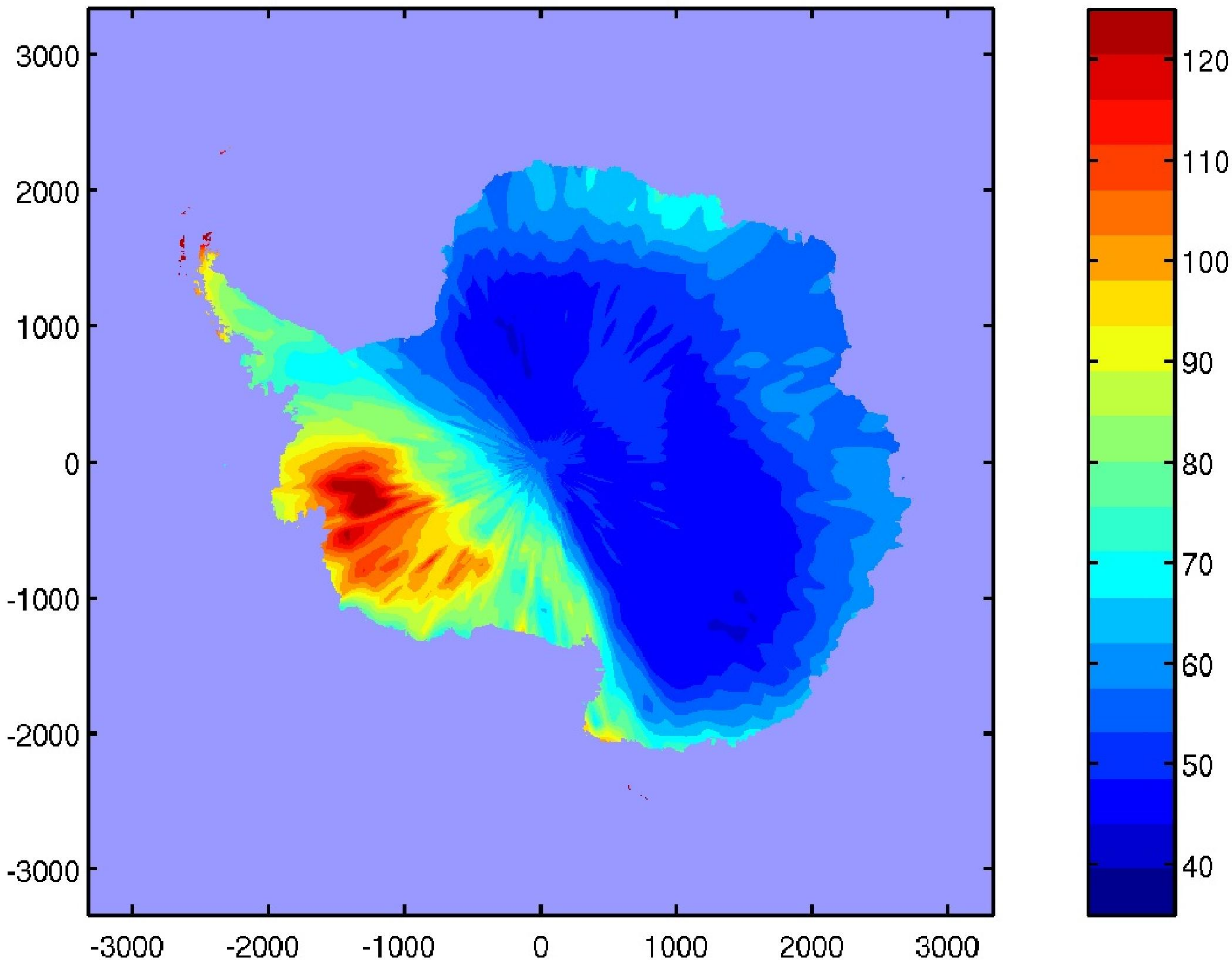
Thickness (m)

British Antarctic Survey 2004



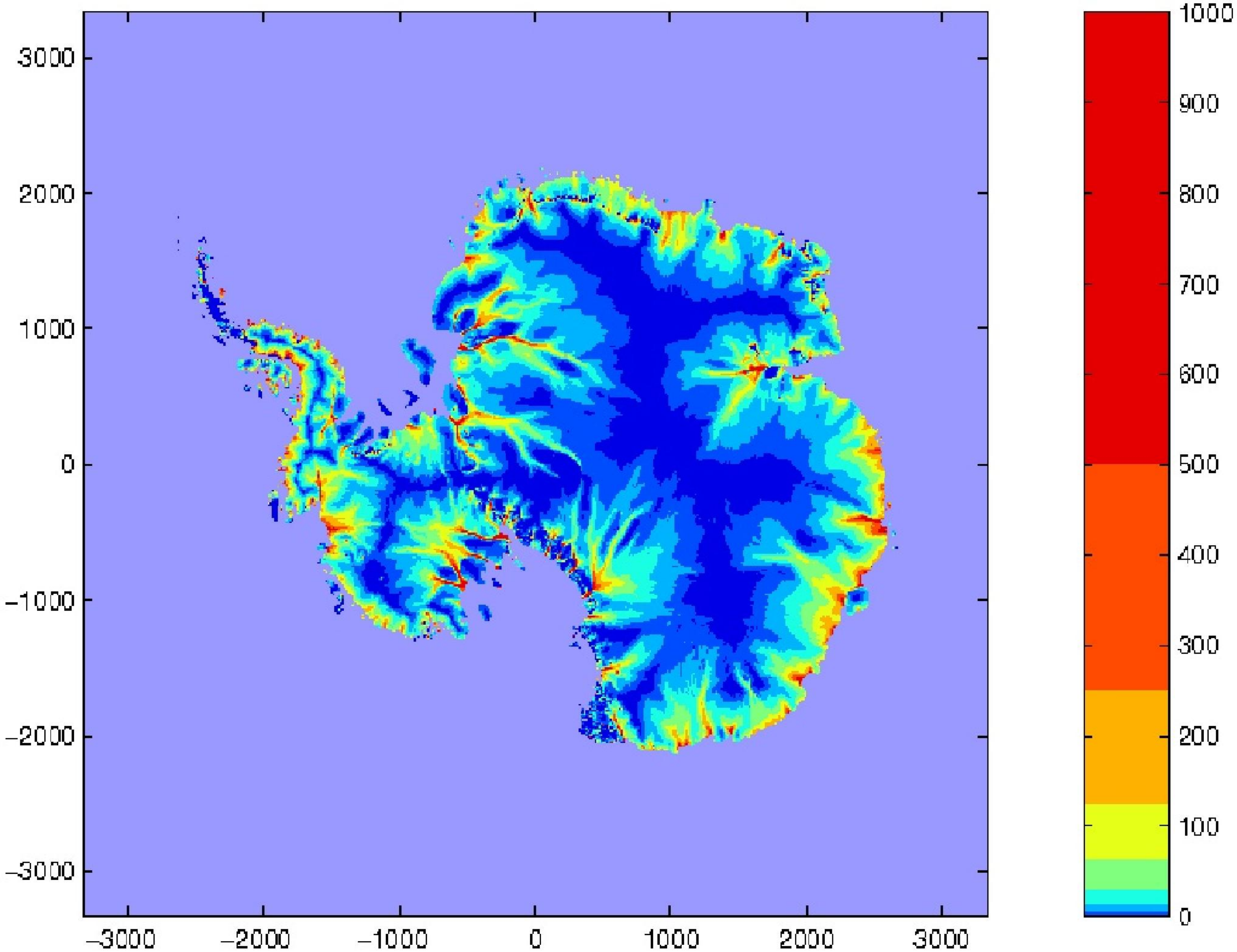
Geothermal flux (mW/m^2)

Shapiro & Ritzwoller (2004; Earth Planetary Sci. Let.)



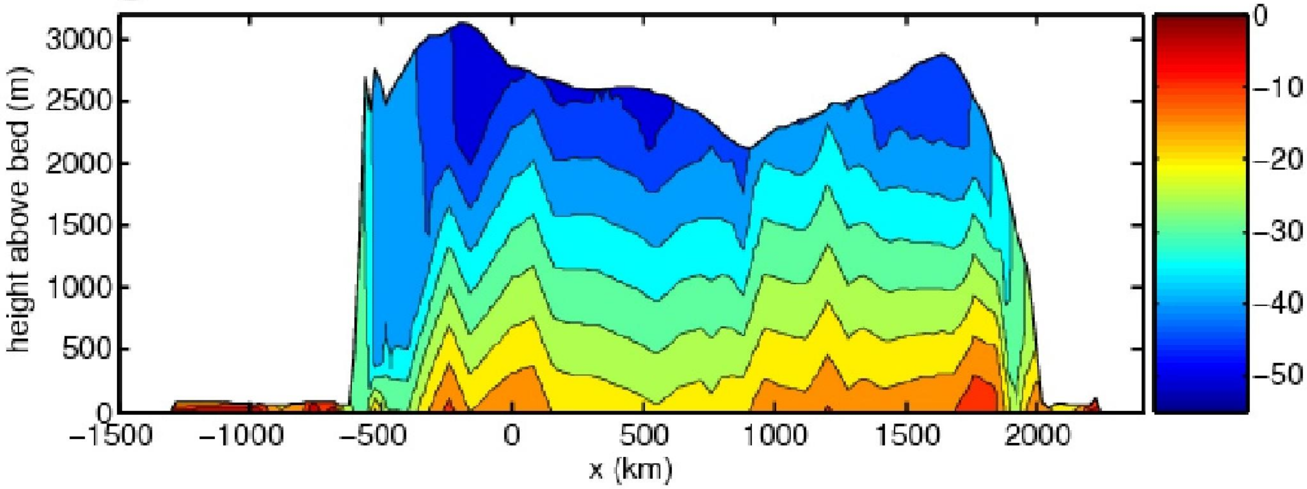
Balance velocity is used for flow mode "mask"

Bamber, Vaughan and Joughin (2000) based on Budd and Warner (1996) algorithm

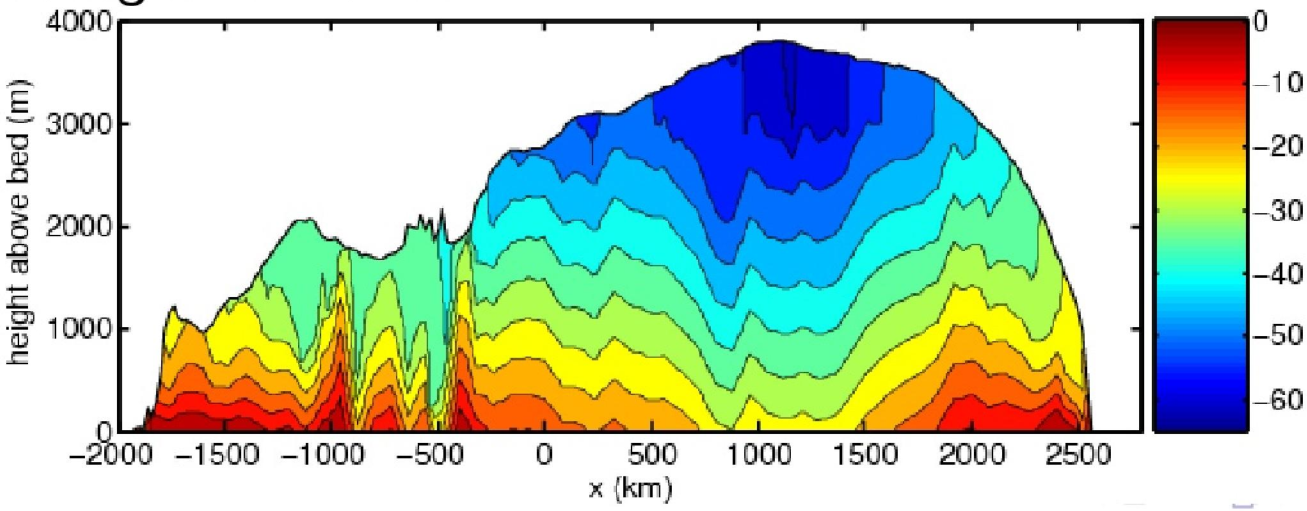


Modeled temp along sections through S pole [*preliminary*]

Along 0°–180°:

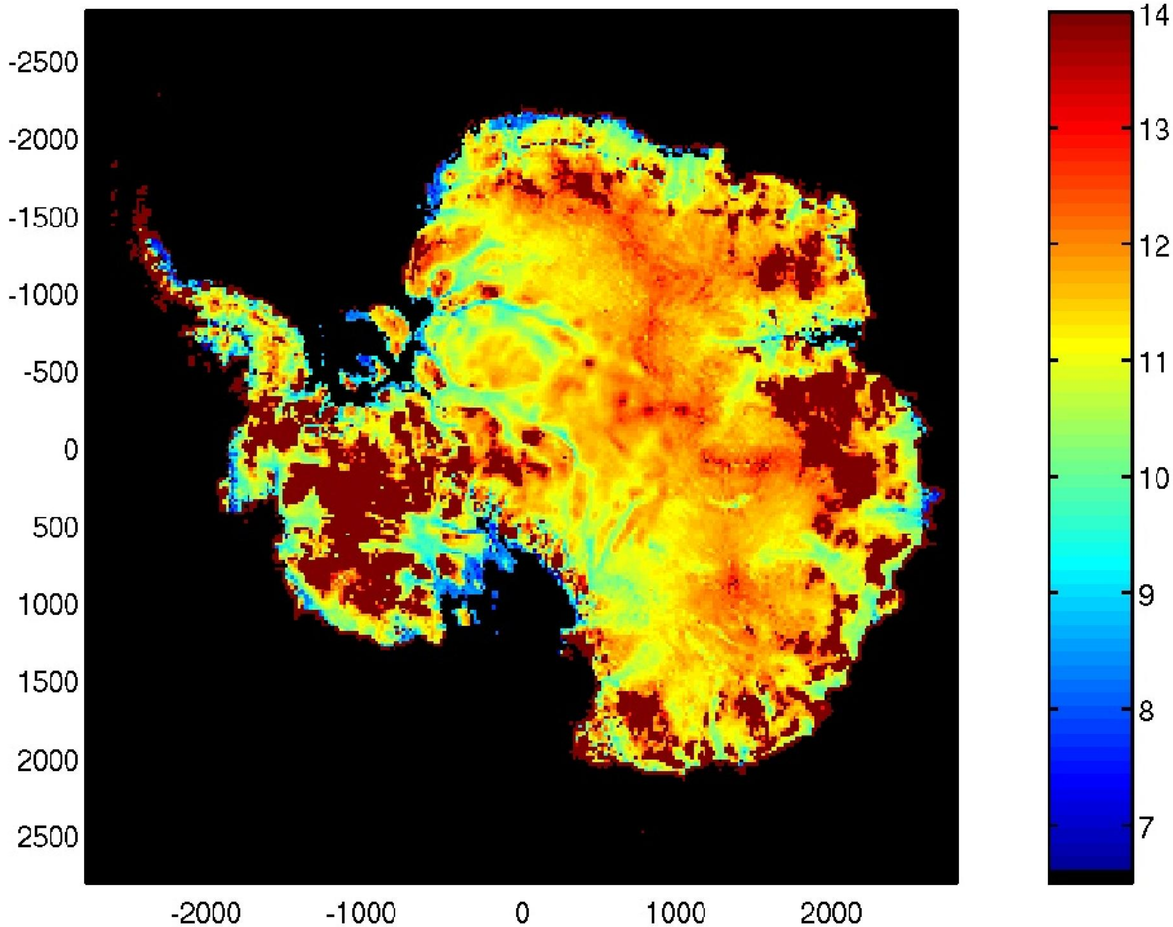


Along 90°W–90°E:



Basal drag assuming linear law: $(\text{stress}) = -\beta \mathbf{U}_{\text{sliding}}$

$\log_{10}(\beta)$ where β is in units Pa s m^{-1}. Compare constant value 2.0×10^9 Pa s m^{-1} in (Hulbe and MacAyeal 1998). *Preliminary.*



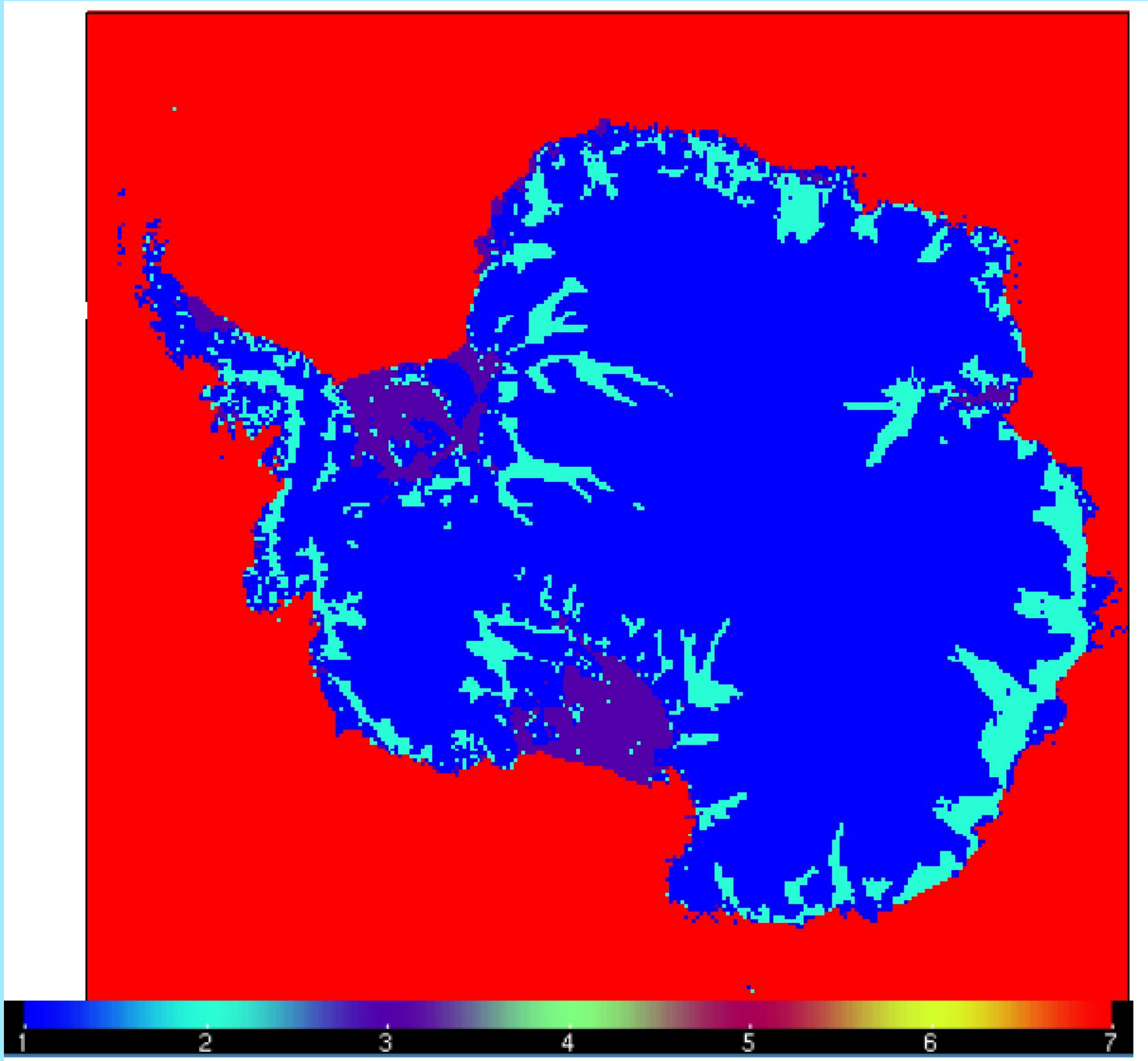
How the last slide was created

Getting basal drag from balance velocities and the SIA

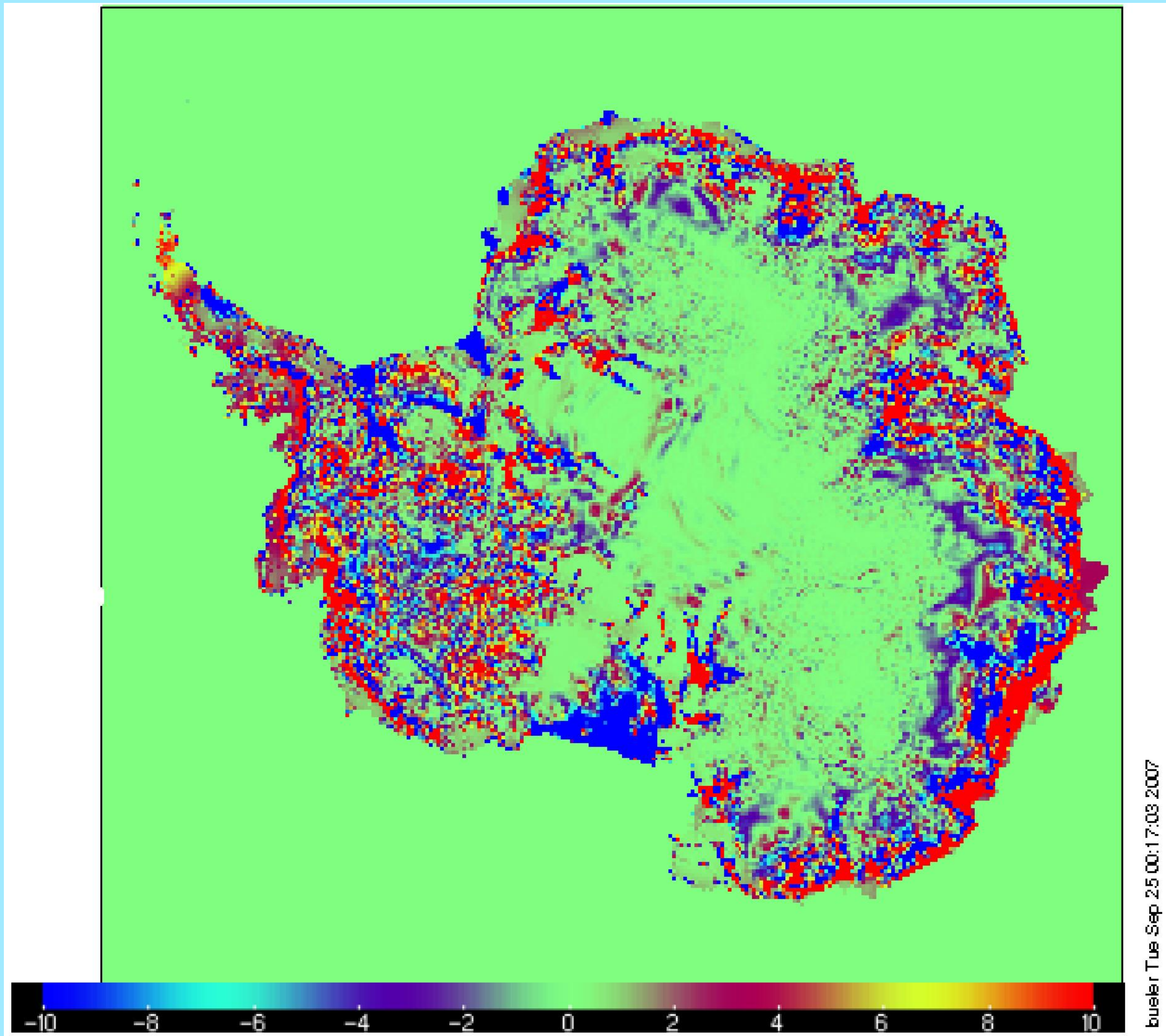
1. deformational (SIA) velocities are computed at all grounded points (using Goldsby-Kohlstedt)
2. average deformational velocity is subtracted from mass-balance velocity to give a sliding velocity
3. this sliding velocity is put into the MacAyeal-Morland equations at all grounded points to determine the drag coefficient which would give this much sliding

Notes

- If deformational velocities exceed balance velocities then get negative drag! Here we set $\beta = 10^{14}$ Pa s m^{-1} in that case.
- Effect of high geothermal flux in Amundsen sector (from Shapiro and Ritzwoller map) is clear.

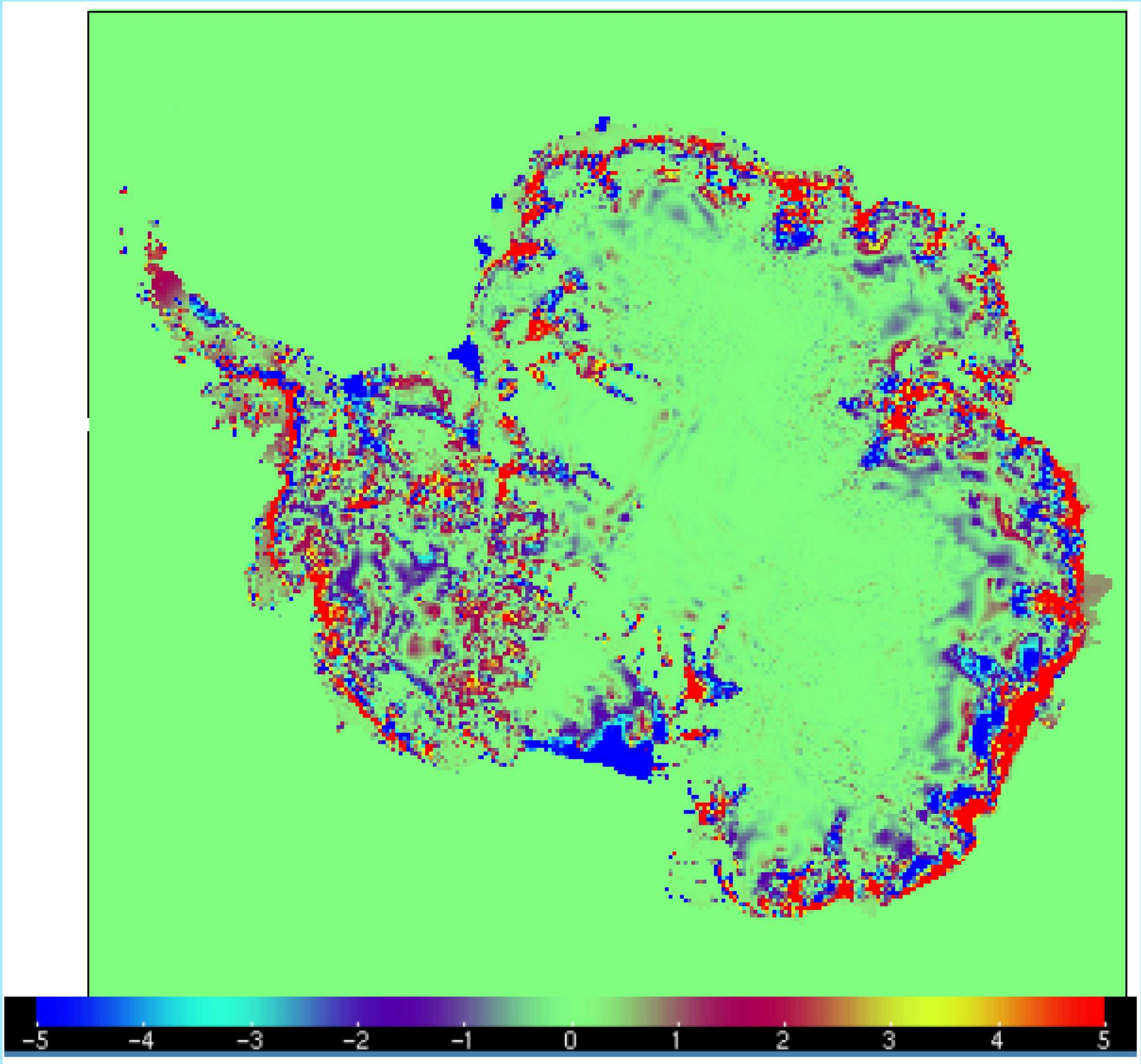


Mask 1=SIA, 2=ice stream, 3=floating ice shelf, 7=ice free ocean



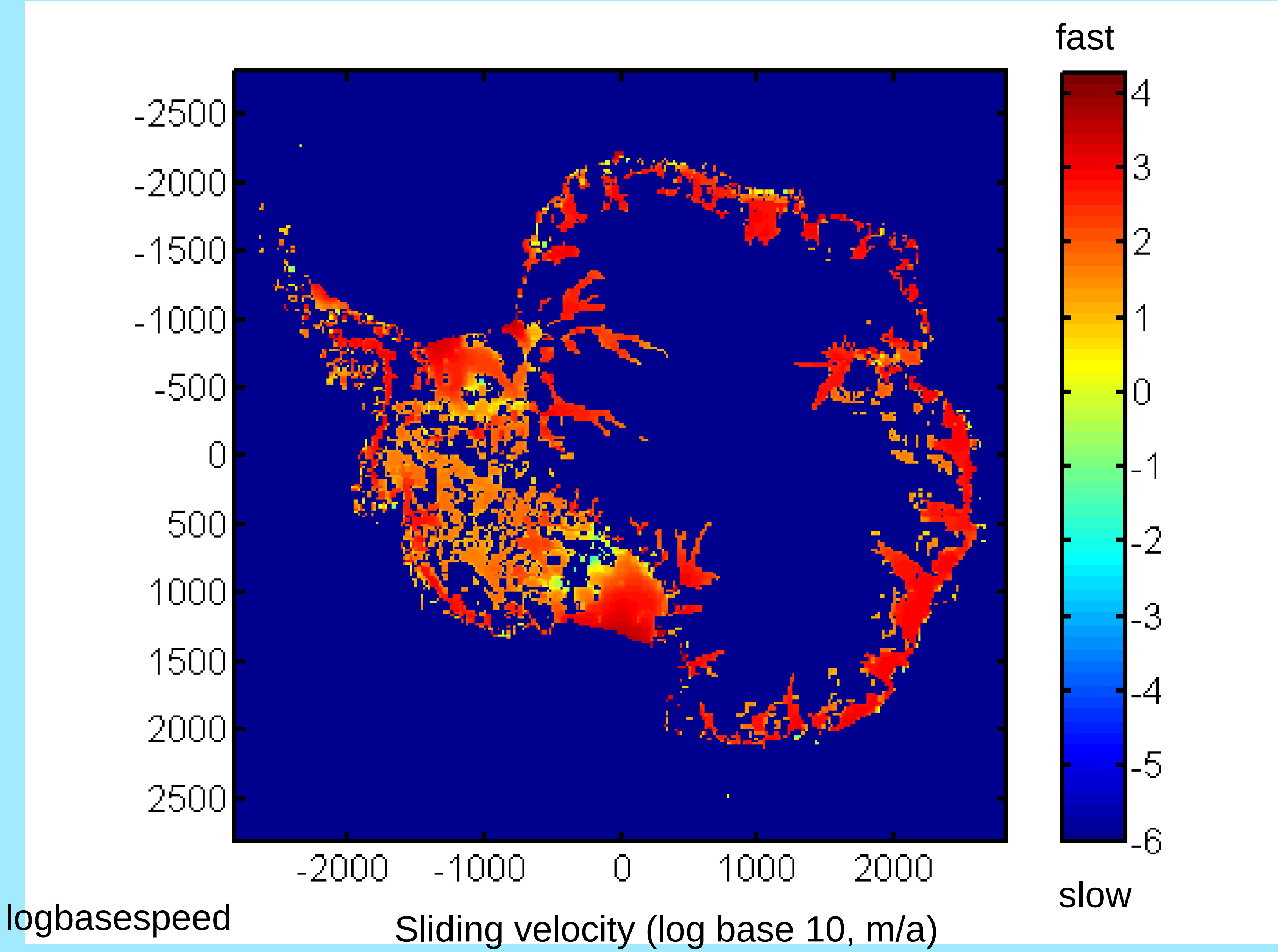
loss gain

dH Total thickness change at end of model run (m)

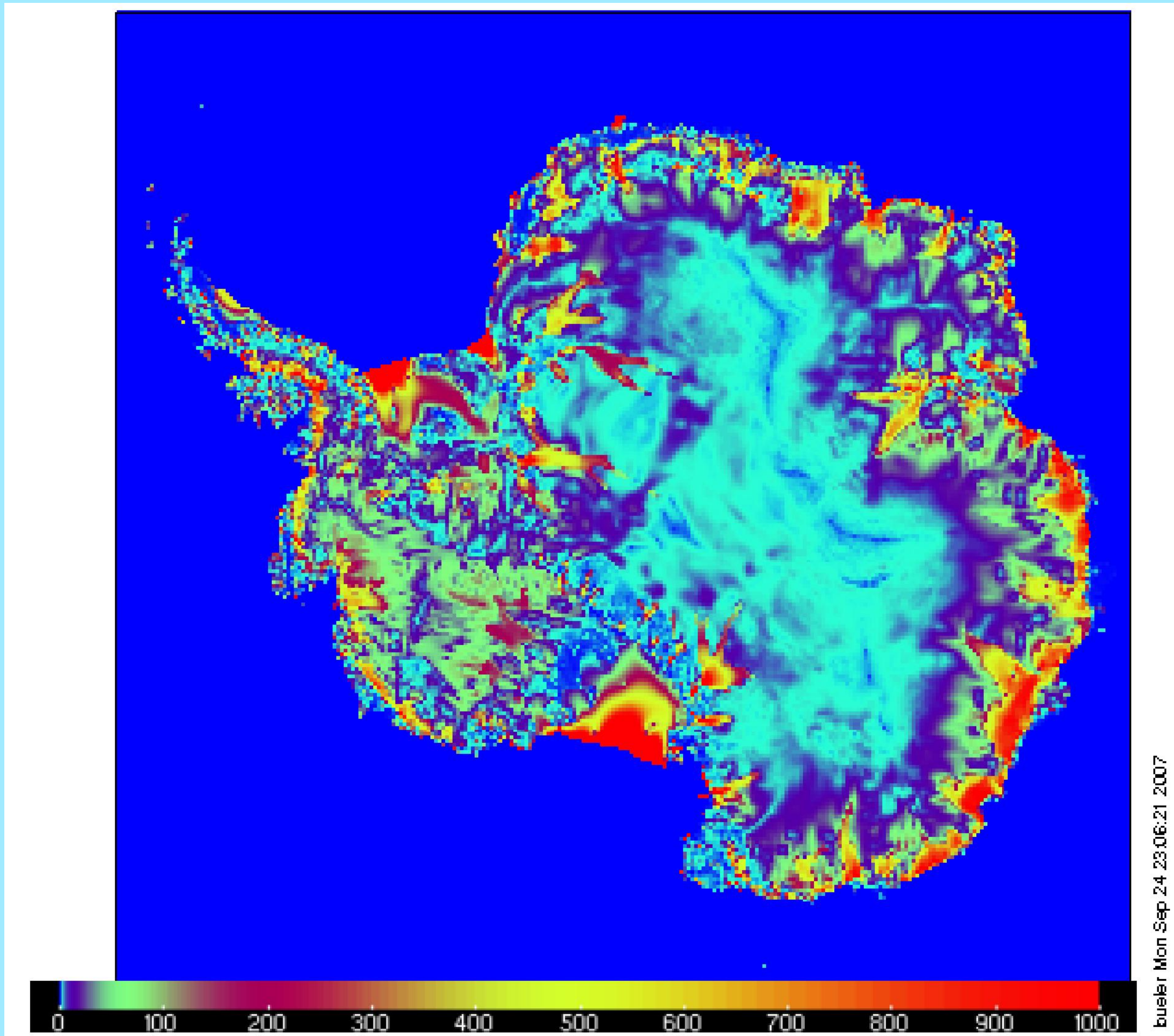


loss gain

dH/dt Time rate of thickness change (m/a)


fast
4
3
2
1
0
-1
-2
-3
-4
-5
-6
slow
-2500
-2000
-1500
-1000
-500
0
500
1000
1500
2000
2500
-2000
-1000
0
1000
2000
logbasespeed
Sliding velocity (log base 10, m/a)

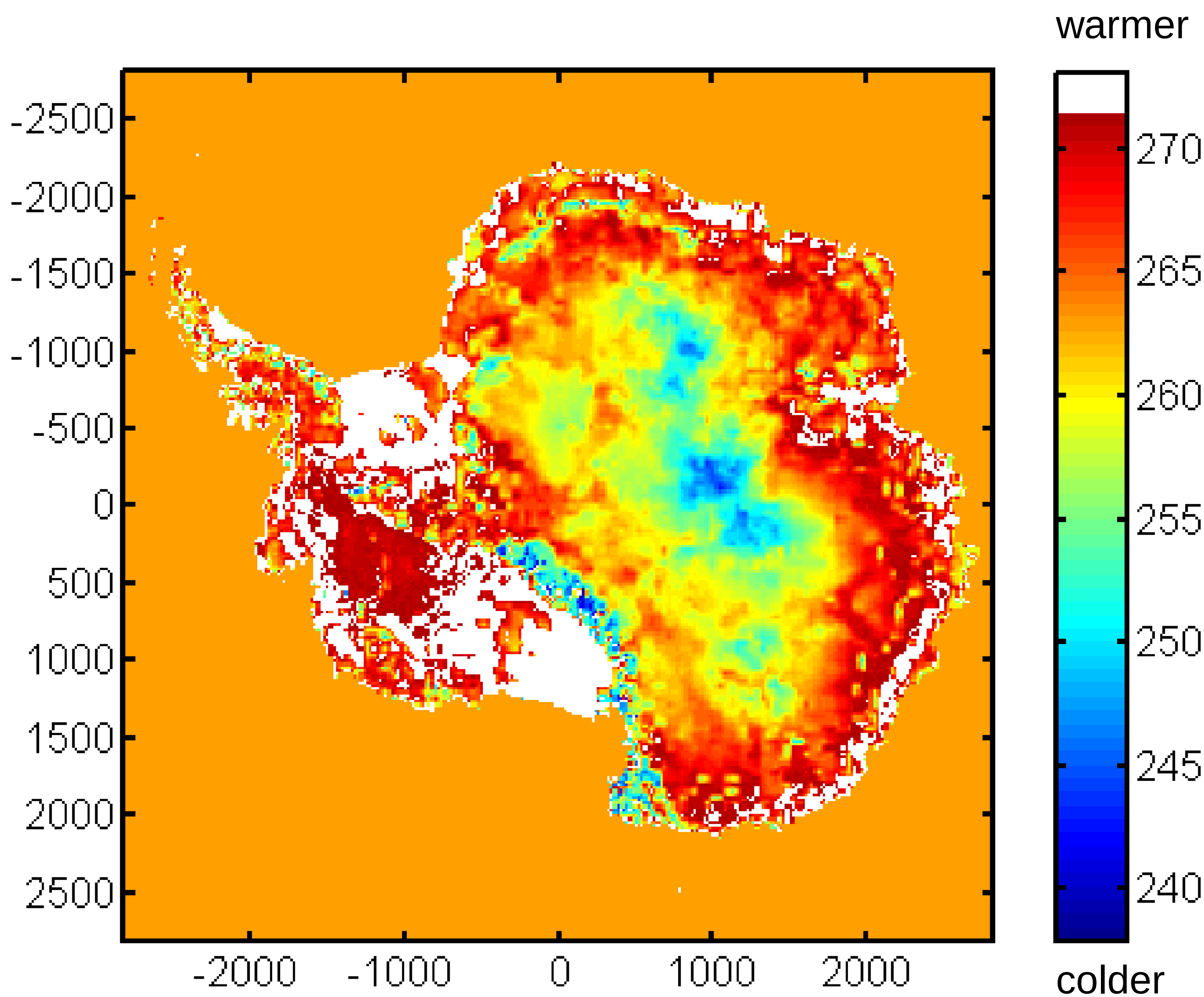
slow



fast

cbar

Vertically-averaged velocity (m/a)



Tbase

Basal temperature (K)