Enthalpy formulations for ice sheet modeling, and the role of basal melt rates

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Outline

what is an "ice sheet model"?

- describes ice as a slow, viscous fluid *forced* only by gravity and ocean interaction:

  geometry and boundary stress and ice strength determine velocity instantaneously

- *inputs* at start time: surface elevation and thickness (geometry), ice temperature (?)
- *boundary inputs*: upper surface mass+energy balance, sub-shelf mass+energy balance, ocean forces, sub-glacial layer strength (?)
- *outputs* (at each time): geometry, velocity, rate of total mass change, isostacy

above lists are

- incomplete
- dependent on user: nature of input/outputs of an ice sheet model depend on the user's intent

Parallel Ice Sheet Model (PISM)

- homepage `www.pism-docs.org`
- *completely* open source
- well-documented for users and developers/modifiers
- in use by:
  - University of Alaska, Fairbanks
  - Centre for Ice and Climate
  - Danish Climate Center at DMI
  - Potsdam Institute for Climate Impact Research (PISM-PIK)
  - Max Planck Institute for Meteorology, Hamburg
  - Institute for Marine and Atmospheric Research, Utrecht
  - Antarctic Research Centre, Victoria University, New Zealand
  - ?? . . . you don't have to tell us you are using PISM!
- SIA and SSA stress balance solutions (= velocity)
  - computed separately, heuristically-combined: SSA-as-a-sliding-law
- thermomechanically-coupled (using enthalpy since stable0.3)
- extensible: well-defined boundary/coupling interfaces

PISM performance

- highly-variable grid resolution depending on application:
  - * Florian Ziemen at MPI, Hamburg: 40 km grid for northern hemisphere paleo-, deglaciation model
  - * Andy Aschwanden at UAF: 40 m grid for Störglaciaren thermodynamics study
- $\Delta t \sim \Delta x^2$ (because some diffusive processes are treated explicitly)
- example parallel performance description:
  - * modest parameter study of whole ice sheet Greenland ice dynamics
  - * ten century-length 3 km runs using 256 processors (Cray XT5 at ARSC UAF)
  - * $\approx$ 300 million (u, v, w, H) unknowns
  - * Δt about 0.02 model-years
  - * total $\approx$ 8000 processor-hours ... not much!
- PETSc

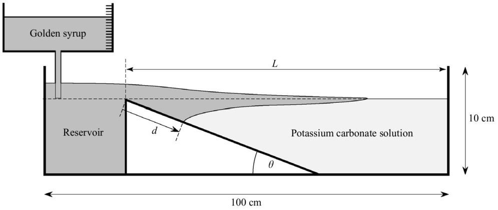
Outline

should your ice sheet model conserve energy?

of course it should! . . . *why?*

counterexample?

figure 7 from Robison et al. (2010)



reasons for including conservation of energy in ice sheet models:

- ice viscosity depends on temperature
- . . . and it depends on liquid water content if temperate
- basal melt rate is computed from heat fluxes at base of ice

but: why should we care about basal melt rate?:

- it dominates subglacial water pressure
- critical unknown for fast grounded ice flow in rapidly-changing climates

two kinds of ice: cold and temperate

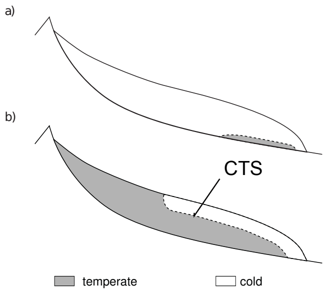
- *cold ice* has temperature below the pressure-melting point and zero/negligible liquid water (in the polycrystaline ice matrix)
- *temperate ice* has temperature equal to the pressure-melting point and positive amount of liquid water

(Andy wants you to know:

on the use of language:

- ice can be either cold or temperate
- . . . and only glaciers can be *polythermal*)

polythermal types



two most commonly found polythermal structures (schematic only): a) Canadian-type and b) Scandinavian-type

- prior work on polythermal ice sheet models: Greve (1995,1997), SICOPOLIS
- consensus 2002–2009 (my perception): *polythermal ice plays minor role . . . marine ice sheets and "full Stokes" are the hot topics!*

thought experiment

consider an idealized drainage basin of Jakobshavn size:

- 300 km by 300 km $\approx 10^5$ km^2 $= 10^{11}$ m^2
- average SMB in basin is 0.2 m/yr
- . . . so this total mass is added per year:

$$m_{SMB} = (10^{11}\,\text{m}^2)(0.2\,\text{m})(1000\,\text{kg}\,\text{m}^{-3}) = 2 \times 10^{13}\,\text{kg}$$

- average surface elevation over the basin is $z_{av} = 2000$ m

(*note on temperature change versus melting*:

- $c_i = 2009$ J/(kg K) but $L = 3 \times 10^5$J/kg is the latent heat of fusion
- . . . L is equivalent to raising temperature of ice 160 degrees)

the thought experiment:

> *In ice sheet steady state mass m_{SMB} appears at calving front at zero elevation, and 2000m gravitational energy has been dissipated. How much ice can you melt with all this energy?*

thought experiment 2

steady state:

- $m_{SMB} = 2 \times 10^{13}$ kg mass
- corresponds to potential energy (using $g = 10\,\text{m}\,\text{s}^{-2}$):

$$\Delta E = m_{SMB} \cdot g \cdot z_{av} = 4 \times 10^{17}\,\text{J}$$

- how much ice could be melted by this much energy? (using $4/3 \approx 1$):

$$L m_{melt} = \Delta E \qquad \Longrightarrow \qquad m_{melt} = \frac{4 \times 10^{17}}{3 \times 10^{5}}\,\text{kg} \approx 10^{12}\,\text{kg}$$

- corresponds to melting this volume in one year:

$$V_{melt} = \frac{10^{12}\,\text{kg}}{1000\,\text{kg}\,\text{m}^{-3}} = 1\,\text{km}^3$$

thought experiment 3

where does the melting happen?

- all this energy is *not* concentrated in one place, but instead as distributed strain-dissipation heating
- *but* it appears in places where strain rates times deviatoric stresses are highest
- . . . e.g. near the base in thick, fast-flowing ice with high surface slopes

interlude (in thought experiment)

when running PISM you may have seen:

- typical 3D cell volumes: $\approx 10\,\text{km} \times 10\,\text{km} \times 20\,\text{m} = 2\,\text{km}^3$
- in transient runs PISM will sometimes report several 3D grid cells are fully-melted, e.g.:

```
PISM WARNING: fully-liquified cells detected:
volume liquified = 58.537 km^3
```

- happens most often with
  - * fine near-base vertical resolution ($\sim$ 5 m)
  - * longish time steps (several years)
  - * there is sliding and something caused change in basal strength
- a solution (if needed) is to shorten the time step

thought experiment 4

how could so much ice be melted in a model time step?

- a big reduction in basal resistance or calving-front force can cause a rapid drop in surface elevation throughout the basin (*observed*, e.g. in Jakobshavn)
- causes margin advance (grounded) or big calving event (tidewater)
- if uniform surface drop of vertical distance Δz occurs in time step Δt then this much energy is dissipated as heating:

$$\begin{aligned}\Delta E &= \left[(10^{11}\,\text{m}^2)(\Delta z\,\text{m})(1000\,\text{kg}\,\text{m}^{-3})\right]\; g\, z_{av} \\ &= \left[10^{14}\Delta z\right]\; g\, z_{av} = 2\times 10^{18}\,\Delta z\ \text{J}\end{aligned}$$

- if $Lm_{melt} = \Delta E$:

$$m_{melt} = \frac{2\times 10^{18}\Delta z}{3\times 10^5}\,\text{kg} \approx 10^{13}\Delta z\ \text{kg},$$

$$V_{melt} = \frac{10^{13}\Delta z\,\text{kg}}{1000\,\text{kg}\,\text{m}^{-3}} = 10\Delta z\ \text{km}^3.$$

thought experiment 5

- from last slide: $V_{melt} = 10\Delta z$ km^3 if surface drops Δz meters in Jakobshavn-size basin
- . . . and this was just one of many basins in a whole Greenland model
- if several Jakobshavn-size basins lose 1 m of surface elevation in a $\Delta t = 1$ year time step then yes, you can melt some 3D grid cells!

- *done with thought experiment*
- *note that* most *basal melt is less extreme*

Outline

concept 1: glacier ice is a mixture of ice and liquid water

- mixture density is sum of partial densities:

$$\rho = \rho_{\mathsf{i}} + \rho_{\mathsf{w}}$$

- liquid water fraction, often called water (moisture) content:

$$\omega = \frac{\rho_{\mathsf{w}}}{\rho}$$

- velocity of mixture ("barycentric velocity"):

$$\rho \mathbf{v} = \rho_{\mathsf{i}} \mathbf{v}_{\mathsf{i}} + \rho_{\mathsf{w}} \mathbf{v}_{\mathsf{w}}$$

- because observed ω are small ($< 3\%$; Petterson et al.,) we treat the mixture as incompressible:

$$\rho \approx \hat{\rho}_{\mathsf{i}}$$

enthalpy generally

- Wikipedia, the source of all truth:

  > *Enthalpy is a measure of the total energy of a thermodynamic system. It includes the internal energy, which is the energy required to create a system, and the amount of energy required to make room for it by displacing its environment and establishing its volume and pressure.*

- that is,

$$H = U + pV$$

  where H is *enthalpy*, U is *internal energy*, p is pressure, and V is the volume of the system

- but we are applying the enthalpy concept to a mixture of incompressible fluids, and for each of these pressure *does no work*
- so, for our application:

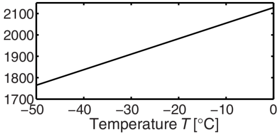
$$H = U$$

  and "enthalpy" is just an abbreviation for "internal energy"

concept 2: enthalpy defineable for solid and liquid

- choose cold temperature, e.g.: $T_0 = 223.15K$ for convenience
- enthalpy for pure ice (figure from Greve & Blatter, 2009):

$$H_{\mathrm{i}} = H_{\mathrm{i}}(T) = \int_{T_0}^{T} C_{\mathrm{i}}(\tilde{T})\,\mathrm{d}\tilde{T}$$



- enthalpy for liquid water:

$$H_{\mathrm{w}} = H_{\mathrm{w}}(T,p) = \int_{T_0}^{T_{\mathrm{m}}(p)} C_{\mathrm{i}}(\tilde{T})\,\mathrm{d}\tilde{T} + L + \int_{T_{\mathrm{m}}(p)}^{T} C_{\mathrm{w}}(\tilde{T})\,\mathrm{d}\tilde{T},$$

- enthalpy for mixture:

$$\rho H = \rho_{\mathrm{i}} H_{\mathrm{i}} + \rho_{\mathrm{w}} H_{\mathrm{w}}.$$

mixture enthalpy

- recall $\omega = \rho_\mathsf{w}/\rho$ so $1 - \omega = \rho_\mathsf{i}/\rho$
- the mixture enthalpy (J / kg) is:

$$H = H(T, \omega, p) = (1 - \omega) H_\mathsf{i}(T) + \omega H_\mathsf{w}(T, p).$$

- define the enthalpy of the cold/temperate ice transition:

$$H_\mathsf{s}(p) = \int_{T_0}^{T_\mathsf{m}(p)} C_\mathsf{i}(\tilde{T})\, \mathrm{d}\tilde{T}$$

- we assume the mixture is always partly ice, and that its liquid water component is always at the pressure-melting point
- then:

$$H(T, \omega, p) = \begin{cases} H_\mathsf{i}(T), & H \le H_\mathsf{s}(p), \\ H_\mathsf{s}(p) + \omega L, & H_\mathsf{s}(p) < H, \end{cases}$$

concept 3: temperature and liquid fraction are functions of enthalpy

- now we undo all of that!
- think: enthalpy is the basic/state variable
- invert the functions
- temperature and liquid fraction are functions of enthalpy and pressure:

$$T(H,p) = \begin{cases} T_{\text{i}}(H), & H \le H_{\text{s}}(p), \\ T_{\text{m}}(p), & H_{\text{s}}(p) < H, \end{cases}$$

$$\omega(H,p) = \begin{cases} 0, & H \le H_{\text{s}}(p), \\ L^{-1}(H - H_{\text{s}}(p)), & H_{\text{s}}(p) < H. \end{cases}$$

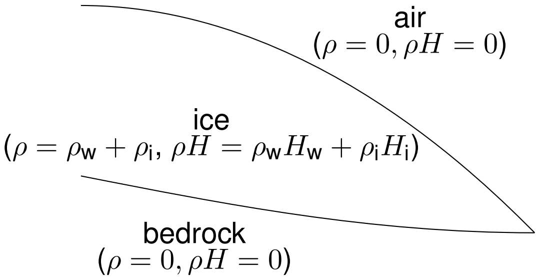
diagram

- diagram above for fixed pressure p
- temperature of mixture is function of enthalpy: $T = T(H, p)$ (solid line)
- also the liquid water fraction: $\omega = \omega(H, p)$ (dotted line)
- at temperature $T_{\mathrm{m}}(p)$:
    - $H_{\mathrm{s}}(p) =$ enthalpy of pure ice
    - $H_{\mathrm{l}}(p) =$ enthalpy of pure liquid water
    - $L = H_{\mathrm{l}}(p) - H_{\mathrm{s}}(p)$

Outline

concept 4: mixture fields defined everywhere

- consider fields ρ and ρH, mixture densities
- these fields are defined in air, ice, and bedrock
- ... but they undergo jumps at the ice upper surface ($z = h$) and the ice base ($z = b$)



general balance

sorry to be abstract but this gets used:

- consider a scalar quantity ψ describing particles (fluid) which move with velocity $\mathbf{v}$ within a region V (e.g. enthalpy or density)
- the *advective flux* is $\psi\mathbf{v}$
- ...but there may be a *non-advective* (e.g. conductive) *flux* $\boldsymbol{\phi}$; same units as $\psi\mathbf{v}$
- then the *balance* of ψ is:

$$\frac{\partial \psi}{\partial t} = -\nabla \cdot (\psi\mathbf{v} + \boldsymbol{\phi}) + \pi$$

  where π is the rate of *production* of ψ
- this balance equation is an Eulerian view of the fluid, as the region V is fixed

concept 5: mixture mass and enthalpy balances follow from general balances on components

as an example consider the ice enthalpy density:

- we formulate separate component energy balances

$$\frac{\partial(\rho_\text{i} H_\text{i})}{\partial t} = -\nabla \cdot (\rho_\text{i} H_\text{i} \mathbf{v} + \mathbf{q}_\text{i}) + Q_\text{i} - \Sigma_\text{w}$$

$$\frac{\partial(\rho_\text{w} H_\text{w})}{\partial t} = -\nabla \cdot (\rho_\text{w} H_\text{w} \mathbf{v} + \mathbf{q}_\text{w}) + Q_\text{w} + \Sigma_\text{w}$$

  where Q_i, Q_w are strain-heating rates in the components and $\Sigma_\text{i}, \Sigma_\text{w}$ are *exchange* rates between components

- conservation of energy for part of mixture: $\Sigma_\text{i} + \Sigma_\text{w} = 0$
- adding and simplifying gives a mixture balance

$$\rho \frac{\text{d}H}{\text{d}t} = -\nabla \cdot \mathbf{q} + Q$$

concept 6: heat flux in ice requires empirical constitutive relation

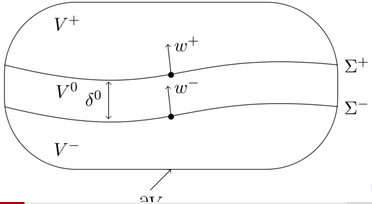
- for cold ice there is conduction by Fourier (mostly upward in slow ice)
- likewise for temperate ice ... but following the gradient of the pressure-melting temperature (usually downward!)
- but in temperate ice the liquid component may be mobile: *empirical relation needed, and few experiments*
- we propose:

$$\mathbf{q} = \begin{cases} -k_\mathsf{i} C_\mathsf{i}(H)^{-1}\,\nabla H, & \text{cold ice,} \\ -k(H,p)\nabla T_\mathsf{m}(p) - k_0 \nabla H, & \text{temperate ice.} \end{cases}$$

- note: conduction is written in terms of enthalpy gradient

concept 7: jump conditions across active layers include thin-layer transport

- consider V^0, a thin firn/runoff layer at top of ice sheet, or a thin subglacial hydrologic layer at base of ice sheet . . . handled the same way!
- surfaces $\Sigma^\pm$ bound the active layer V^0
- standard jump conditions ($[\![\psi(\mathbf{v}\cdot\mathbf{n} - w_\sigma)]\!] + [\![\boldsymbol{\phi}\cdot\mathbf{n}]\!] = 0$) apply on $\Sigma^\pm$
- below is a "pillbox" including such a thin active layer V^0 in which scalar ψ is advected and produced
- take the $\delta^0 \to 0$ limit of the general balance
- surfaces $\Sigma^\pm$ converge to a single surface σ



jump conditions across active layers, cont.

- the result is *both* a jump condition and a thin layer balance:

$$[\![\psi(\mathbf{v}\cdot\mathbf{n}-w_\sigma)]\!] + [\![\boldsymbol{\phi}\cdot\mathbf{n}]\!] + \frac{\partial\lambda_\sigma}{\partial t} + \nabla\cdot\left(\lambda_\sigma\mathbf{v}_\sigma + \boldsymbol{\phi}_\sigma\right) = \pi_\sigma.$$

- for example, if there is runoff at the ice upper surface, a layer liquid water of variable thickness η_{r}, and if we define

$$M_{\mathsf{r}} = -\frac{\partial(\rho_{\mathsf{w}}\eta_{\mathsf{r}})}{\partial t} - \nabla\cdot\left(\rho_{\mathsf{w}}\eta_{\mathsf{r}}\mathbf{v}_{\mathsf{r}}\right)$$

  as the mass balance from runoff, then
- this new "'jump condition" simplifies to a form of the *surface kinematical equation*:

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x} + v\frac{\partial h}{\partial y} - w = \rho^{-1}N_h(a^\perp + M_{\mathsf{r}})$$

basal melt rate is described the same way

consider the enthalpy density ρH; how does it jump at the ice base?:

- let

$$M_b = -\frac{\partial(\rho_{\text{w}}\eta_b)}{\partial t} - \nabla\cdot\left(\rho_{\text{w}}\eta_b\mathbf{v}_b + \mathbf{\Phi}_b\right), \tag{1}$$

$$Q_b = -\frac{\partial(\rho_{\text{w}}H_{\text{w}}\eta_b)}{\partial t} - \nabla\cdot\left(\rho_{\text{w}}H_{\text{w}}\eta_b\mathbf{v}_b + \mathbf{\Psi}_b\right). \tag{2}$$

  these are rates at which mass, enthalpy (respectively) are delivered by subglacial transport to a location on the ice base
- let $F_b = \mathbf{v}\cdot(\mathbf{T}\cdot\mathbf{n})$, the rate of friction heating
- then the jump condition is:

$$M_b H + (\mathbf{q} - \mathbf{q}_{\text{lith}})\cdot\mathbf{n} = F_b + Q_b. \tag{3}$$

  where $\mathbf{q}, \mathbf{q}_{\text{lith}}$ are non-advective (conductive) heat fluxes in ice and bedrock respectively

summary of enthalpy formulation concepts

1. glacier ice is a mixture of ice and liquid water
2. enthalpy is defineable for solid and liquid
3. temperature and liquid fraction are functions of enthalpy
4. mixture fields ρ and ρH are defined everywhere
5. mixture mass and enthalpy balances follow from general balances on components
6. heat flux in ice requires empirical constitutive relations (Fourier is not enough!)
7. jump conditions across active layers include thin-layer transport

PISM's enthalpy formulation

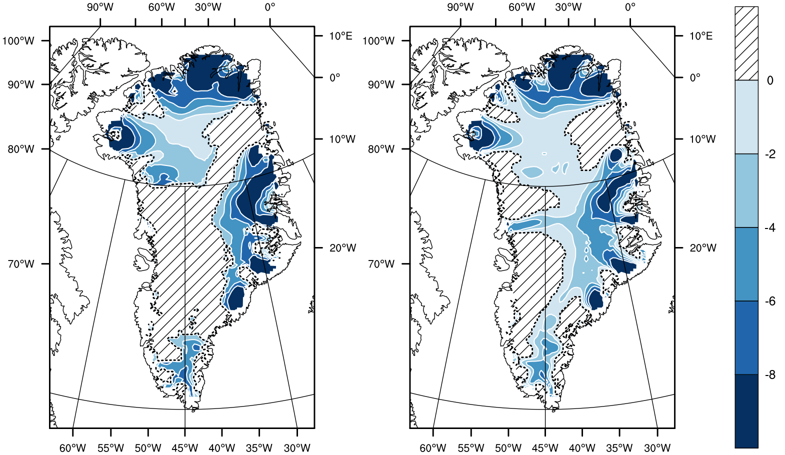
PISM takes these concepts and implements them imperfectly in a

- shallow
- finite difference
- parallel

framework

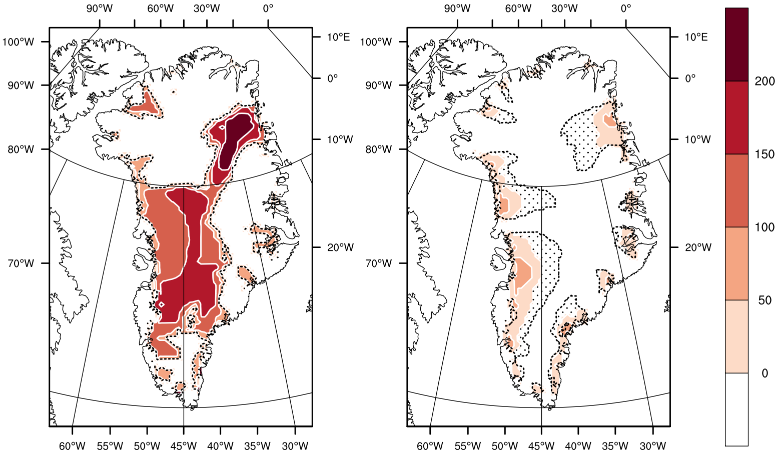
Outline

basal temperature



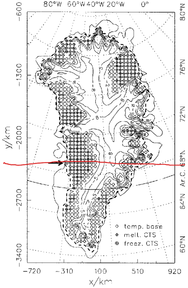
Pressure-adjusted temperature (°C) at the base for the control run (left) and the cold-mode run (right). Hatched area = basal ice is temperate.

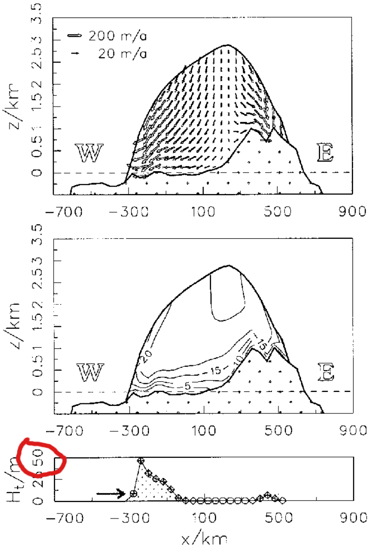
temperate ice



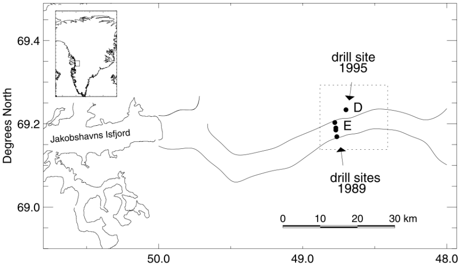
Thickness (m) of the basal temperate ice layer for the control run (left) and the cold-mode run (right).

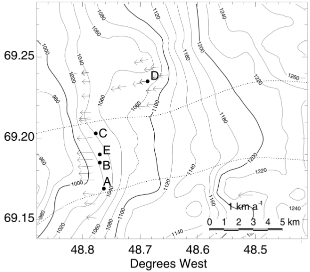
compare to model results by Greve (1997)

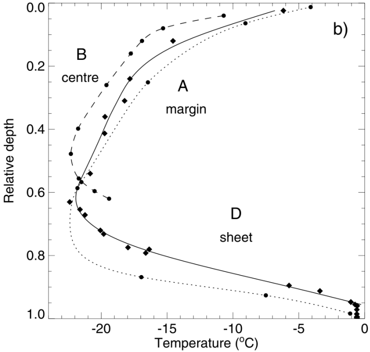




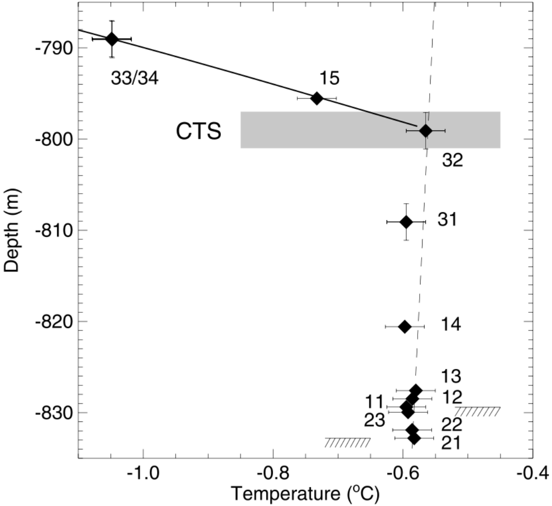
borehole temperatures in fast ice: Lüthi et al 2002





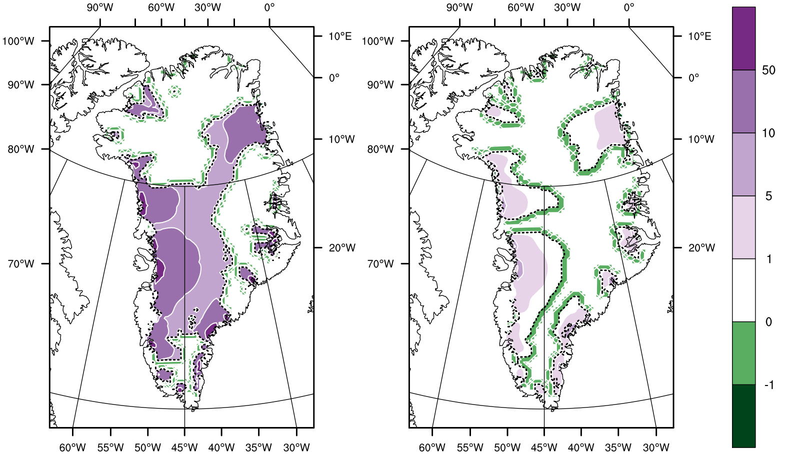


borehole temperatures in fast ice 2: Lüthi et al 2002



detail at borehole D "sheet"

basal melt rate: significant to fast ice dynamics!



Basal melt rate (mm/year) for the control run (left) and the cold-mode run (right). Negative values indicate freeze-on.