How ice sheets flow, and how to model it on a computer

Ed Bueler

(with help from Andy Aschwanden and Constantine Khroulev)

Dept. of Mathematics and Statistics
and Geophysical Institute
UAF

19 October 2012

Outline

how do ice sheets flow?

ice sheet models do what?

progress and challenges

questions?

Outline

how do ice sheets flow?

ice sheet models do what?

progress and challenges

questions?

ice in glaciers is a *viscous fluid*

- mostly

ice in glaciers is a *viscous fluid*

- (ice sheets are just big glaciers)
- we describe fluids primarily by a *velocity field* $\mathbf{u}(t, x, y, z)$
- if the ice fluid were
  - faster-moving, and
  - linearly-viscous

  then ice flow would be a "typical" fluid like liquid water
- we would use the Navier-Stokes equations as our flow model:

$$\nabla \cdot \mathbf{u} = 0 \qquad \textit{incompressibility}$$

$$\rho \left(\mathbf{u}_t + \mathbf{u} \cdot \nabla \mathbf{u}\right) = -\nabla p + \nu \nabla^2 \mathbf{u} + \rho \mathbf{g} \qquad \textit{force balance}: \; ma = F$$

- so, to numerically model our glacier fluid, do we grab a textbook on computational fluid dynamics (CFD) and go?

is numerical ice flow modeling a part of CFD?

- yes
- large scale like atmosphere/ocean
- . . . but it is a weird one
- consider what makes atmosphere/ocean modeling exciting:
    - turbulence
    - convection
    - coriolis force
    - density variation
- none of the above is relevant to ice flow
- so what could be interesting about the flow of slow, cold, stiff, laminar, old ice?
- it's "*ice dynamics!*"

ice is a slow, shear-thinning fluid

- our glacier fluid is

  slow: $\rho\left(\mathbf{u}_t + \mathbf{u}\cdot\nabla\mathbf{u}\right) \approx 0$

  non-Newtonian: viscosity ν is not constant
- "slow":

$$\rho\left(\mathbf{u}_t + \mathbf{u}\cdot\nabla\mathbf{u}\right) \approx 0 \qquad \iff \qquad \begin{pmatrix}\text{forces of inertia} \\ \text{are negligible}\end{pmatrix}$$

- "non-Newtonian": flow is "shear-thinning", so larger strain rate means smaller viscosity
- thus the standard ice flow model is Glen-law ($n = 3$) Stokes:

$$\begin{aligned} \nabla\cdot\mathbf{u} &= 0 && \textit{incompressibility} \\ 0 &= -\nabla p + \nabla\cdot\tau_{ij} + \rho\mathbf{g} && \textit{slow force balance} \\ \mathbf{D}u_{ij} &= A\tau^2\tau_{ij} && \textit{Glen flow law} \end{aligned}$$

equations above are true at every instant

because ice is a slow fluid . . .

- because ice is a slow fluid:

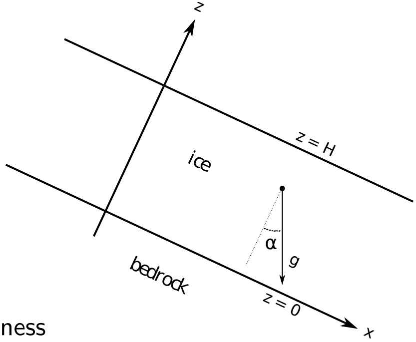
  geometry, boundary stress, and ice viscosity determine velocity field instantaneously

- a time-stepping ice sheet code recomputes the velocity field at every time step, without requiring velocity from the previous step[1]
- thus no memory of previous momentum/velocity
- velocity is a "diagnostic" output of an ice flow model

[1] to be a weatherman you've got to know which way the wind blows . . . but don't expect that much from a glaciologist

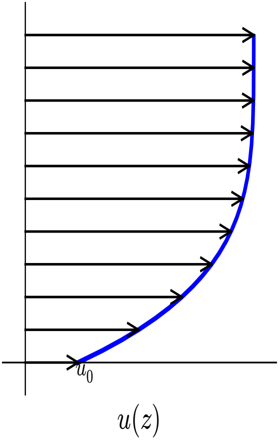
slab-on-a-slope

- an easiest case!
- solve the “standard ice flow model” in a tilted slab, below

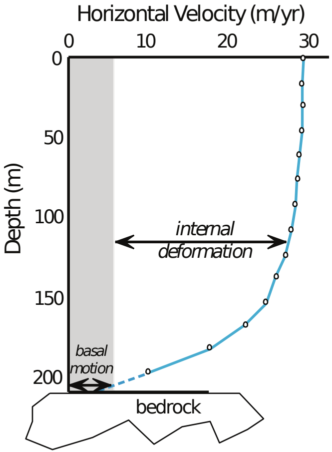


- assume
  - constant thickness
  - no variation in flow with x
- compute velocity $\mathbf{u}(z)$ . . . formulas suppressed

slab-on-a-slope



velocity from slab-on-a-slope formula



velocity profile of the Athabasca Glacier from inclinometry
(Savage and Paterson, 1963)

deformation versus basal motion

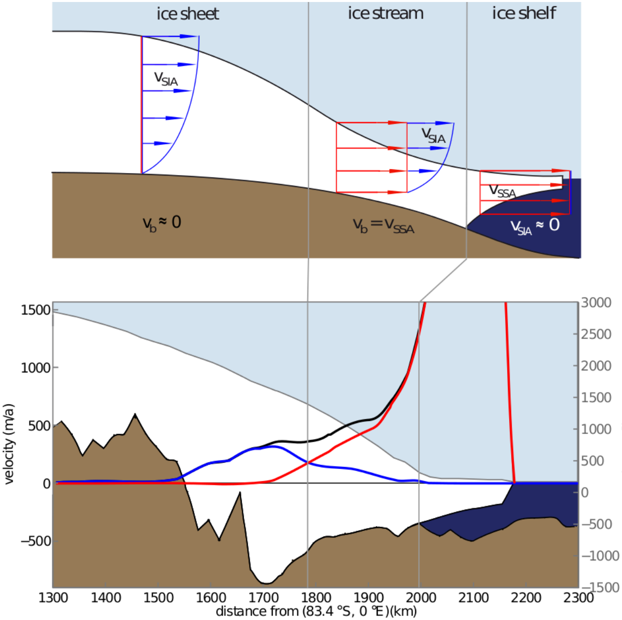
- top:

  cartoon of non-sliding (SIA) and sliding/floating (SSA) modes

- bottom:

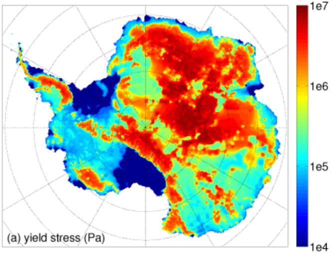
  sheet-stream-shelf transition,

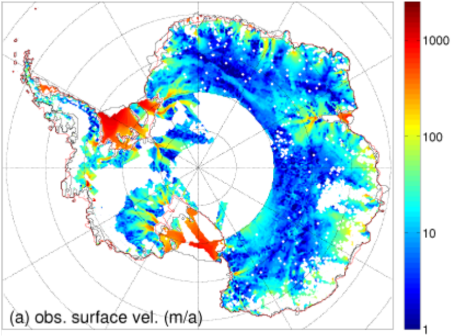
  Lambert Glacier & Amery Ice Shelf, Antarctica

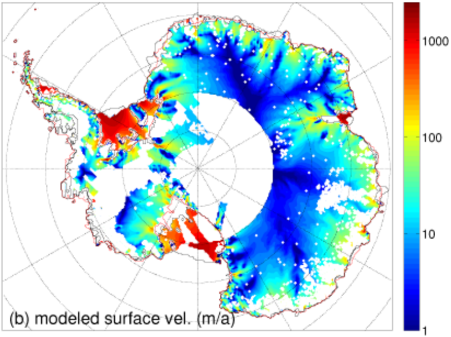


Antarctica is a *marine ice sheet*

- in fact we should not forgetting floating parts of ice sheets
- i.e. *ice shelves*
- and they often have fast upstream grounded ice: *ice streams*





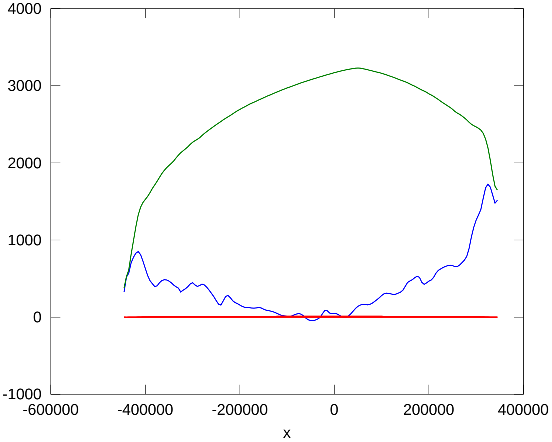


slow, non-Newtonian, some basal slip, and shallow

- ice sheets have four outstanding properties *as fluids*:
  1. slow
  2. non-Newtonian
  3. shallow
  4. contact slip (sometimes)

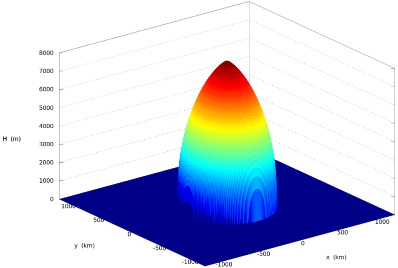
regarding "shallow"

- consider cross section of Greenland ice sheet at 71° N
- below in red is a no-vertical-exaggeration view
    - green and blue: standard vertically-exaggerated cross section

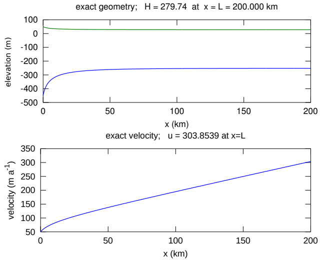


shallow models of ice sheets and shelves

- we don't actually use the "standard ice flow model" (i.e. the Stokes equations) very often
- shown are two most-common shallow approximations
  - top: time-dependent exact solution to the "SIA" = shallow ice approximation
  - bottom: steady exact solution to the "SSA" = shallow shelf approximation
- ...but I'll suppress the partial differential equations for the SIA and SSA models in this talk

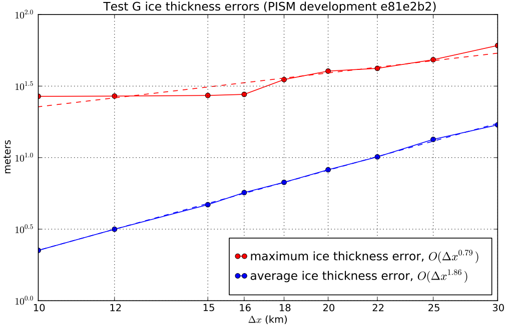


frames from $t = 4$ months to $t = 10^6$ years, equal spaced in *exponential* time



importance of verification

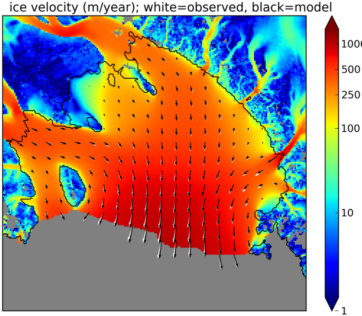
- suppose we are now ice sheet modellers, the chosen few ...
- we take the SIA, SSA, etc. equations and turn them into computer programs
- ... and get pretty pictures
- but last slide showed *exact* solutions
- instead of "eyeballing" we can *measure* errors from the numerical code, as at right

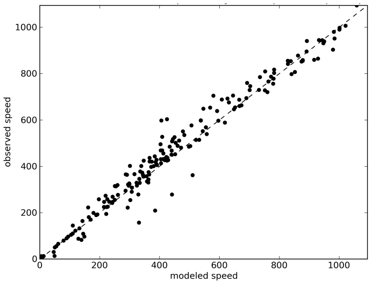


from now on in this talk, I'll assume we have a verified ice sheet model in hand

next step: validation?

- sometimes observational data is
  - of high quality
  - measures exactly what the model is simulating
- for example, below:
  - observed surface velocities versus
  - velocity computed by SSA model in PISM





Outline

ice sheet “weather” forecasting 101

Because ice sheets change more slowly than the atmosphere, predicting their behavior over the coming century has more in common with short-term weather prediction: small errors in the initial state could systematically affect a forecast throughout the 21st century.

(Arthern & Gudmundsson, 2010, J. Glaciol)

ice sheet "weather" forecasting 101

- *weather model testing*: Enter measured forcing variables into a weather forecast model. If the model accurately shows weather events that are known to have occurred then it can be considered successful.

From wikipedia

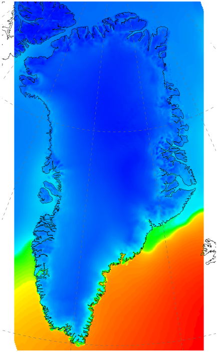
A hindcast is a way of testing a mathematical [prediction] model. Known or closely estimated inputs for past events are entered into the model to see how well the output matches the known results.

- hindcast *before* forecast
- verification *before* (hindcast + validation) *before* forecast

climate "forcings" for a model of an ice sheet

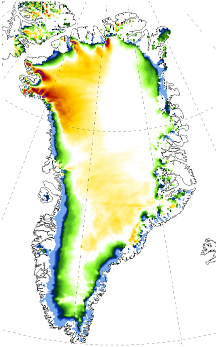
- reanalysis from a regional climate model (HIRHAM5) as climate forcing
- timeseries from 1989–2011 with monthly values of:

2m air temperature



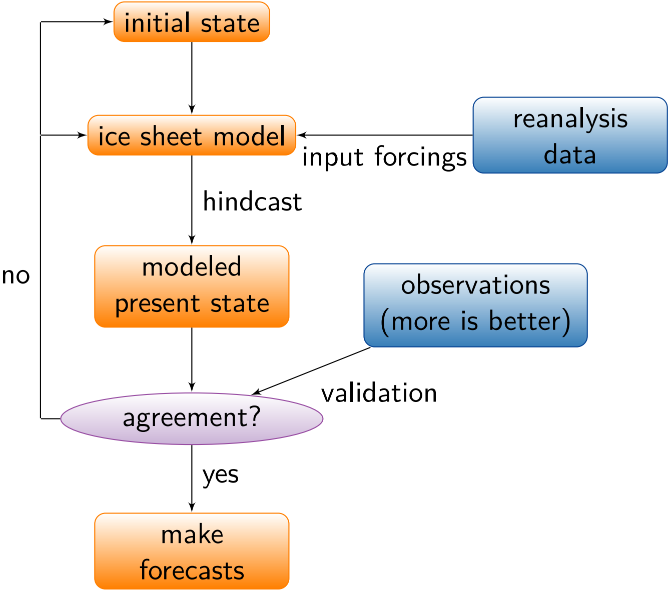
climatic mass balance

(= precipitation minus removal by melting)



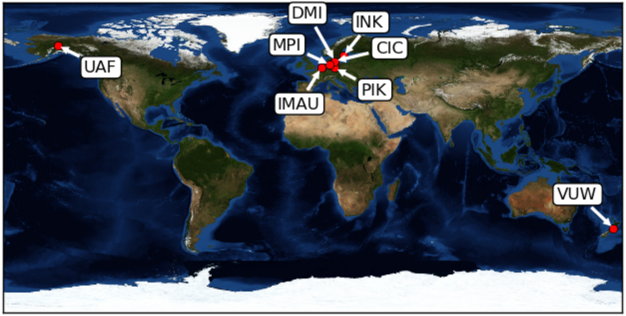
- also: ocean temperatures, geothermal heat, bedrock topography, ...

testing ice sheet initial states



PISM = Parallel Ice Sheet Model

- arguably the most widely-used ice sheet model in the world:



- developed here at UAF
- supported by NASA MAP and ARSC
- see www.pism-docs.org
- . . . but just an example for this talk

generating initial states using PISM

some initialization schemes:

- constant-climate steady-state using present-day climate
- paleo-climate uses (imperfect) data from a full Ice Age cycle
- flux-corrected paleo-climate combines paleo-climate with information about present-day ice thickness

- next four slides: Andy's Greenland runs using PISM on 2 km grid

validation metric: ice volume and ice thickness

- the most common validation metric is ice volume
- ice volume measurement based on ice thickness observation
- PISM Greenland runs comparison:

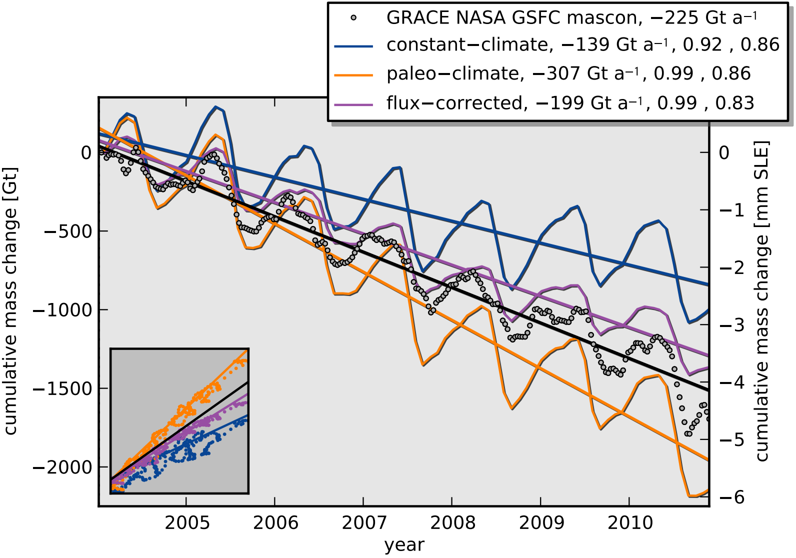
	observed	constant-climate	paleo-climate	flux-corrected
ice volume				
initial volume $[10^6\ km^3]$	2.93	3.18	3.37	X
ice thickness				
avg abs. difference [m]		99	121	X
rms difference [m]		199	244	X

observed ice thickness is from Griggs & Bamber (unpublished)

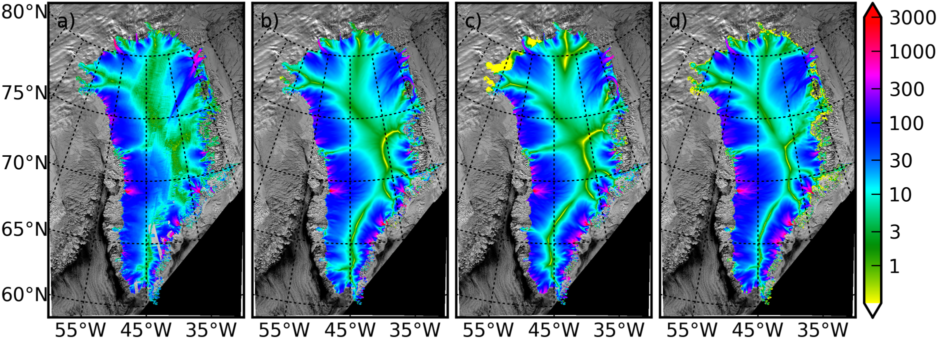
X = ice thickness used in "flux-correcting" is not available for validation

- thus: volume is a weak metric because it averages out positive and negative thickness errors
- how well do we know ice thickness?

validation metric: gravimetric total mass changes



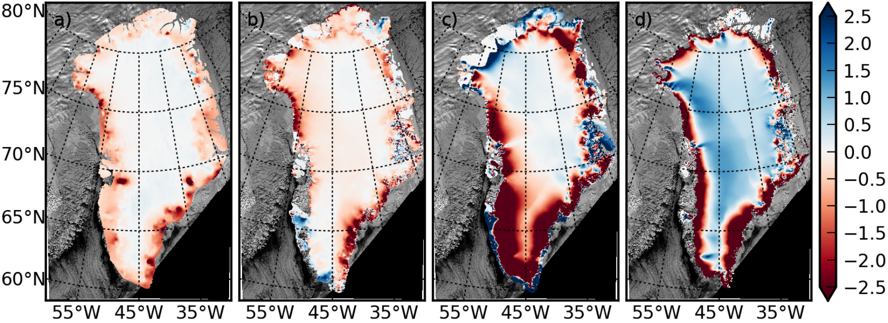
validation metric: surface speeds



a) observed; b) constant-climate; c) paleo-climate; d) flux-corrected

- values in m/a
- observed = interferometric SAR + feature-tracking (Joughin et al., 2010)
- some "data assimilation techniques" (= inverse modelling of the observed velocities) give much better match to observed velocities . . . but it's not clear if time-evolution is better

validation metric: surface elevation change



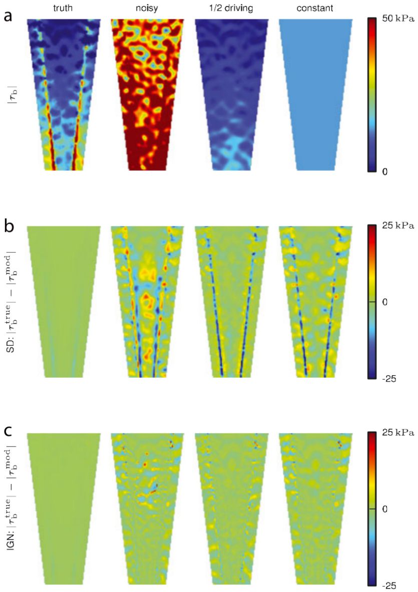
a) observed; b) constant-climate; c) paleo-climate; d) flux-corrected

- values in m
- change over period 2003–2009
- observed = ICESat laser altimetry (Sørensen, 2011)

Outline

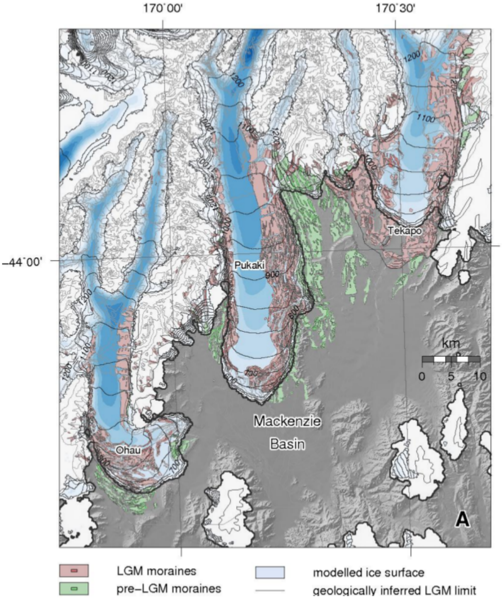
do we know the basal resistance under an ice sheet?

- no
- to slightly better approximation, at times like the present where we know surface velocities, we can invert the ice flow model for basal shear stress
- (in forward mode, the ice flow model turns basal resistance into surface velocity)
- at right: figure from Habermann et al (2012)



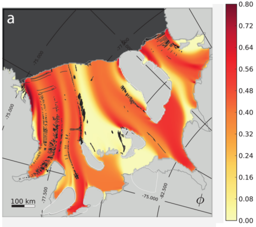
can we effectively use paleo- constraints?

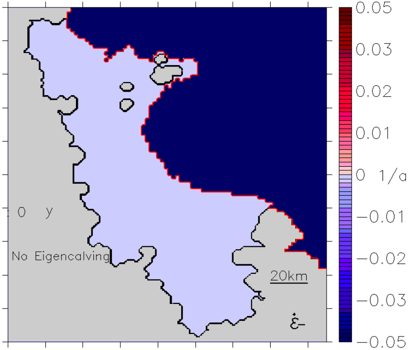
- some of the best information about underneath ice sheets is from geomorphology
- for example, at right is comparison of the LGM moraines of the New Zealand (South Island) ice cap versus a 500 m resolution PISM simulation (Golledge et al., 2012)
- major goal here: recover the climate at the LGM



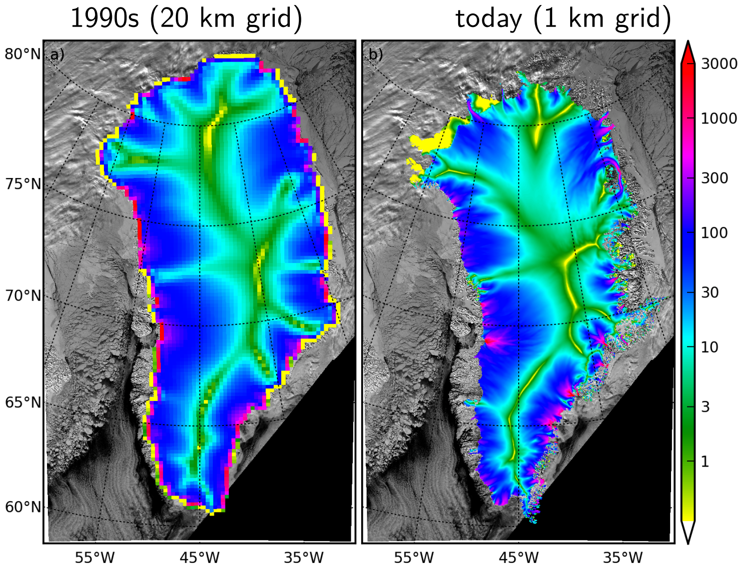
a decent calving law for ice shelves?

- two issues:
  - physical fracture process which causes weakening
  - stress condition at front which causes calving
- top: PISM fracture-density model of the Filchner-Ronne ice shelf showing observed surface crevasse fields (black) and modelled density (color) Albrecht and Levermann (2012)
- top: PISM “eigen-calving” model; modeled steady states of Larsen A & B ice shelves closely-approximate observed Levermann et al. (2012)





we've come a long way, baby?



Outline

how do ice sheets flow?

ice sheet models do what?

progress and challenges

questions?