Superposition of velocity for ice flow modeling

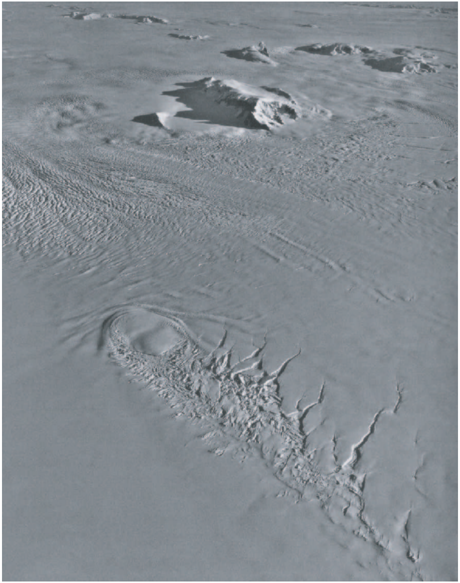
Jed Brown[1], Ed Bueler[2]

[1]VAW, ETH Zürich
[2]Dept. of Mathematical Sciences, University of Alaska Fairbanks

2008-04-18

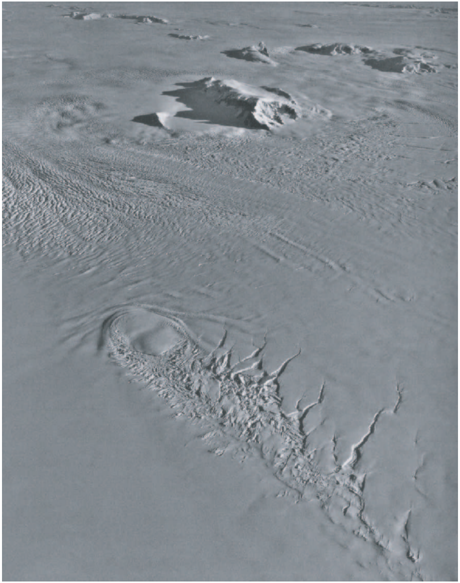
How Shallow?

- Inland ice sheet
  - Small aspect ratio $\epsilon \approx 10^{-3}$
  - Very little sliding



How Shallow?

- Inland ice sheet
  - Small aspect ratio $\epsilon \approx 10^{-3}$
  - Very little sliding
- Outlet glaciers and ice streams
  - Usually still "shallow"
  - Constrained by geometry
  - Slipperyness at the bed varies
  - Flow is not "shallow"



Models

- Stokes
  - Must solve implicit system in 3D
  - Saddle point/poorly conditioned
- Higher order models
  - Still a 3D implicit system, but fewer degrees of freedom
- Shallow Ice and Shallow Streams
  - Only a 2D implicit system
  - Every point is *either* stream or not sliding
  - Margin singularity and wrong physics

What can we do without solving an implicit 3D system?

- General idea
  - Solve a stream-type system for basal, mean, or surface velocity
  - Use SIA-type estimate to produce 3D velocity field
  - Recent work on depth integrated models (Schoof, Hindmarsh)

What can we do without solving an implicit 3D system?

- General idea
  - Solve a stream-type system for basal, mean, or surface velocity
  - Use SIA-type estimate to produce 3D velocity field
  - Recent work on depth integrated models (Schoof, Hindmarsh)
- Practical method
  - Just add them!
  - Naïve method gives too much vertical shear in stream regions
  - Reduce "SIA" component when sliding is easy

Sliding

- SIA sliding is bad

$$\boldsymbol{v} = f(\rho g H \nabla h)$$

  - Discontinous horizontal velocity $\implies$ unbounded vertical velocity
- Power law sliding: $\boldsymbol{\tau}_b = \gamma(\boldsymbol{v}_b)^{\frac{m-1}{2m}} \boldsymbol{v}_b$
- Plastic sliding: $\boldsymbol{\tau}_b = \tau_c \boldsymbol{v} / |\boldsymbol{v}|$
- All sliding comes from stream-type system

Specifics

Shallow Stream equations

$$[2\overline{\eta}H(2u_x + v_y)]_x + [\overline{\eta}H(u_y + v_x)]_y + \tau_{b,x} = \rho g H h_x$$

$$[2\overline{\eta}H(u_x + 2v_y)]_y + [\overline{\eta}H(u_y + v_x)]_x + \tau_{b,y} = \rho g H h_y$$

where $\overline{\eta}$ is depth averaged effective viscosity

$$\overline{\eta} = \overline{B(\theta, \dots)}(\epsilon + \gamma(\boldsymbol{u})/\gamma_0)^{\frac{n-1}{2n}}$$

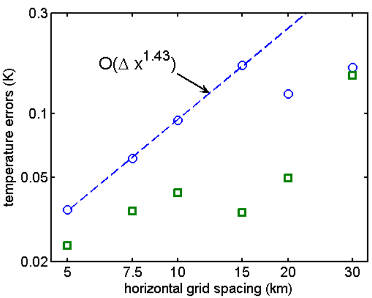
- Combine with shallow ice

$$\boldsymbol{v} = \boldsymbol{v}_{\mathsf{SSA}} + (1 - \frac{2}{\pi}) \tan^{-1}(|\boldsymbol{v}_{\mathsf{SSA}}|^2 / v_0^2)\boldsymbol{v}_{SIA}$$

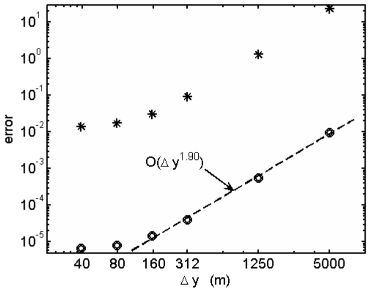
Algorithm

1. Compute provisional SIA velocities everywhere
2. Preprocess: fictitious ice in the ocean, periodic boundary conditions
3. Iteratively solve for basal sliding using SIA inflow conditions for the sliding region
4. Postprocess: remove fictitious ice, combine velocities
5. Geometry/temperature time step

Verification of components

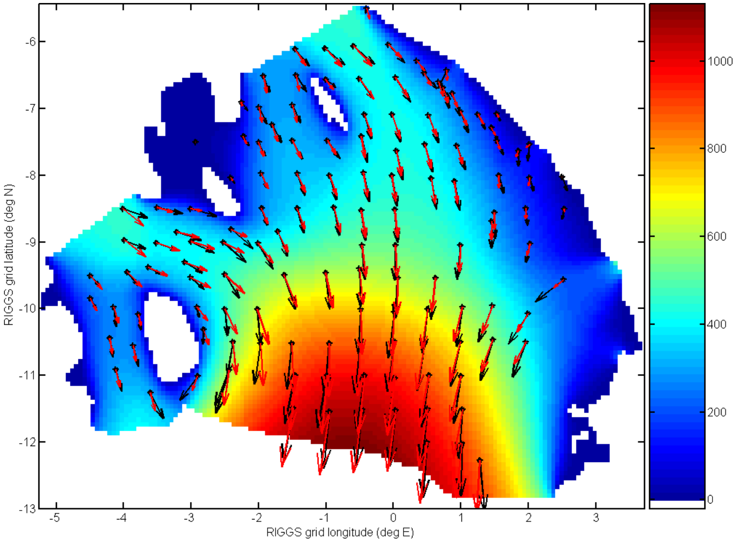


- Thermocoupled shallow ice
- cirles = mean error
- boxes = dome error



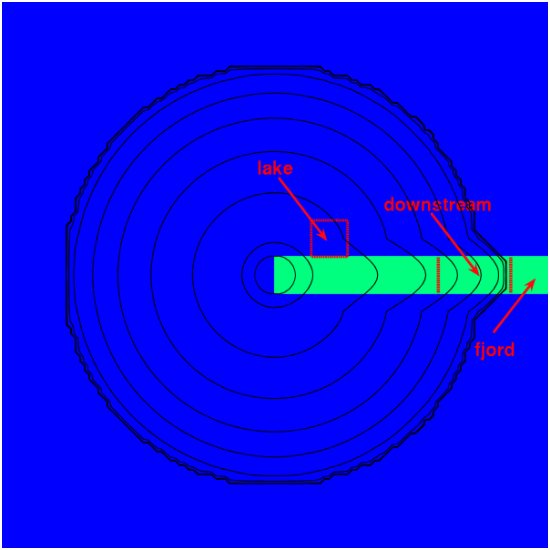
- Shallow streams
- stars = maximum velocity error
- circles = mean relative error

Validation



- 6.8 km grid
- **RIGGS 1983**
- **PISM**

Simple ice cap setup

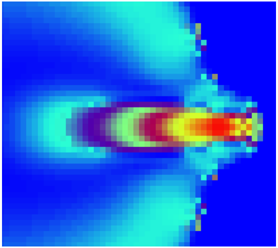


- Till yield stress

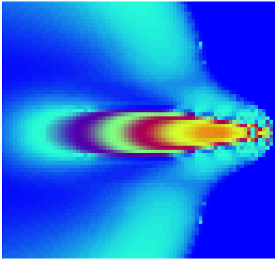
$$\tau_c = (\rho g H - p_w) \tan \phi$$

- $\phi = 20°$
- $\phi = 5°$

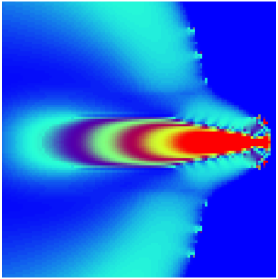
Result



12.5 km



7.5 km



5 km

- 5000 model years after start of sliding
- 100 km wide slippery region, but ice stream narrows dynamically

PISM

- Scalable: 1 billion degrees of freedom, 500 processors
- Constitutive relations: power law, Goldsby-Kohlstedt, mixed
- Flow: Integrated flux
- Thermodynamics: high resolution, basal water layer
- Visco-elastic earth
- Many verification tests
- Greenland and Antarctica at high resolution (5km)
- Open source https://gna.org/projects/pism
- Documentation and user's manual http://pism-docs.org

Rheology

Isotropic viscous fluid

$$\boldsymbol{D} = F(\dots)\boldsymbol{\tau} \quad \text{or} \quad \boldsymbol{\tau} = \eta(\dots)\boldsymbol{D}$$

- Power law: $F = A\,|\boldsymbol{\tau}|^{n-1}$
- Goldsby-Kohlstedt

$$F(\sigma,\theta,P,d) = F_{\mathsf{diff}}(\theta,d) + F_{\mathsf{disl}}(\sigma,\theta,P) + \left(\frac{1}{F_{\mathsf{basal}}(\sigma,\theta)} + \frac{1}{F_{\mathsf{gbs}}(\sigma,\theta,P,d)}\right)^{-1}$$

Rheology

Isotropic viscous fluid

$$\boldsymbol{D} = F(\dots)\boldsymbol{\tau} \quad \text{or} \quad \boldsymbol{\tau} = \eta(\dots)\boldsymbol{D}$$

- Power law: $F = A\,|\boldsymbol{\tau}|^{n-1}$
- Goldsby-Kohlstedt

$$F(\sigma,\theta,P,d) = F_{\mathsf{diff}}(\theta,d) + F_{\mathsf{disl}}(\sigma,\theta,P) + \left(\frac{1}{F_{\mathsf{basal}}(\sigma,\theta)} + \frac{1}{F_{\mathsf{gbs}}(\sigma,\theta,P,d)}\right)^{-1}$$

Rheology

Isotropic viscous fluid

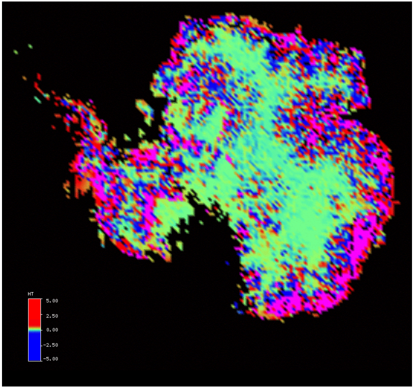
$$\boldsymbol{D} = F(\dots)\boldsymbol{\tau} \quad \text{or} \quad \boldsymbol{\tau} = \eta(\dots)\boldsymbol{D}$$

- Power law: $F = A\,|\boldsymbol{\tau}|^{n-1}$
- Goldsby-Kohlstedt

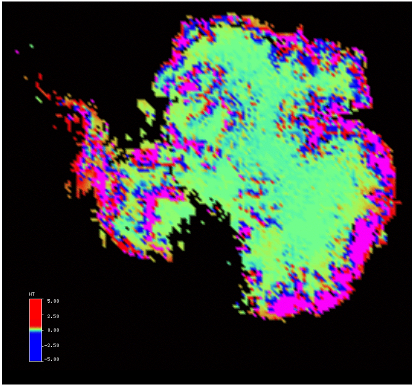
$$F(\sigma,\theta,P,d) = F_{\mathsf{diff}}(\theta,d) + F_{\mathsf{disl}}(\sigma,\theta,P) + \left(\frac{1}{F_{\mathsf{basal}}(\sigma,\theta)} + \frac{1}{F_{\mathsf{gbs}}(\sigma,\theta,P,d)}\right)^{-1}$$

- Commonly used: $\eta = B(\dots)(\epsilon + \gamma_D/\gamma_0)^{\frac{n-1}{2n}}$

Comparison



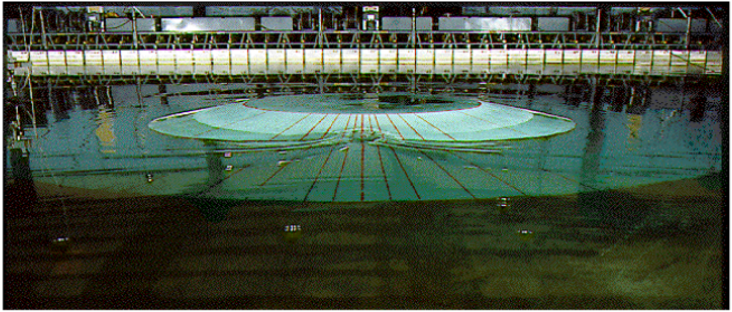
Optimized Glen



Optimized G-K

Experiments

- Verification
  - How well does the numerical model approximate the continuum equations
  - Require exact solutions of the continuum equations
- Validation
  - How well does the model represent reality
  - Need to know about reality



Outlook

- PISM
  - Improved dynamics
  - Inverse modeling
- Full models
  - Must *resolve* outlet glaciers and grounding lines
    - Lots of adaptivity
    - Many degrees of freedom
  - Fully iterative solution
    - Preconditioning
    - Boundary conditions