A model of an ice sheet with one ice stream

(formerly: *How do we actually solve the equations for ice shelves and streams?*)

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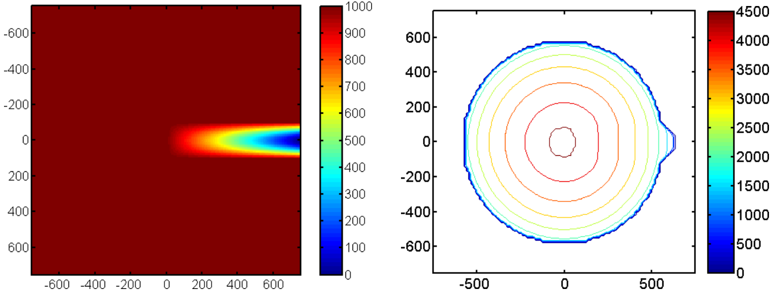
EISMINT2 experiment I

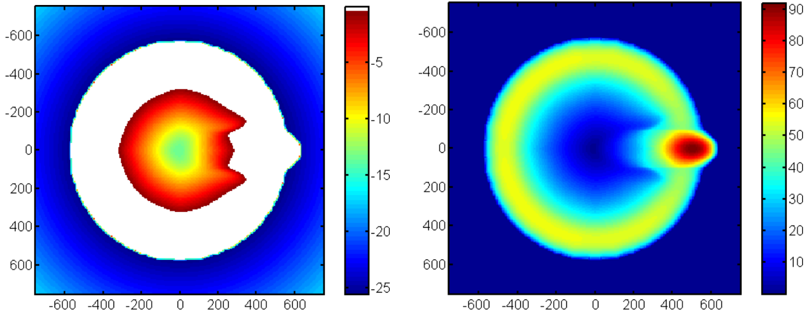


Left: bed elevation (m) *Right*: surface elevation (m) at end of 200ka run

ice flow model is non-sliding thermocoupled shallow ice approximation (SIA)

EISMINT2 report (Payne et al 2000) says about experiment I: "straightforward results with little variability between groups", but results also "difficult to interpret"

EISMINT2 experiment I, cont.



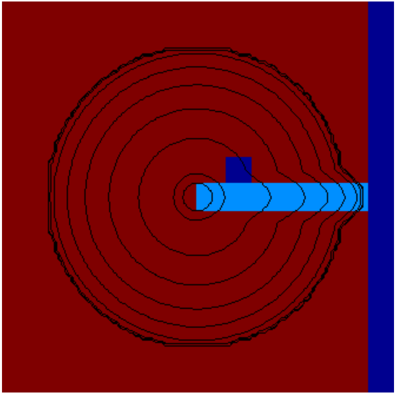
Left: homologous basal temperature (°C) at end of 200ka run; white areas at pressure-melting

Right: vertically-integrated horizontal speed (m/a) at end of run

there "ought" to be an ice stream

but the nonsliding thermocoupled SIA is not the model to predict it

Add plastic till ...



red is $\phi = 20°$
light blue is $\phi = 5°$
dark blue is $\phi = 0°$

"lake" is dark blue square; only present in experiment P4 below

color shows till friction angle ϕ; contours of ice thickness overlain

till yield stress is computed by $\tau_c = (\tan\phi)(\rho g H - p_w)$

(H=ice thickness, p_w = pore water pressure in till [HOW COMPUTED?]; (Paterson, 1994) has this till yield stress model)

"Sliding law" is shallow shelf approximation (SSA)

- will use Schoof's (2006, *A variational approach to ice stream flow*, J. Fluid Mech.) form of "dragging" SSA (MacAyeal, 1989):

$$\frac{\partial}{\partial x_1}\left[2\overline{\nu}H\left(2\frac{\partial v_1}{\partial x_1}+\frac{\partial v_2}{\partial x_2}\right)\right]+\frac{\partial}{\partial x_2}\left[\overline{\nu}H\left(\frac{\partial v_1}{\partial x_2}+\frac{\partial v_2}{\partial x_1}\right)\right]+\tau_{b,1}=\rho g H\frac{\partial h}{\partial x_1},$$
$$\frac{\partial}{\partial x_1}\left[\overline{\nu}H\left(\frac{\partial v_1}{\partial x_2}+\frac{\partial v_2}{\partial x_1}\right)\right]+\frac{\partial}{\partial x_2}\left[2\overline{\nu}H\left(\frac{\partial v_1}{\partial x_1}+2\frac{\partial v_2}{\partial x_2}\right)\right]+\tau_{b,2}=\rho g H\frac{\partial h}{\partial x_2}.$$

  as "sliding law" (*more details later*)

- Schoof's SSA is a free boundary problem; *where* there is sliding and *how fast* are determined simultaneously
- emergent ice streams
- sliding flow is controlled by balance of "membrane" stresses
- vertical plane shear from (SIA) still present; it is added back (*details later*)

(by contrast, *traditional sliding laws have sliding velocity a function of driving shear stress, switched on when base reaches pressure-melting temperature*)

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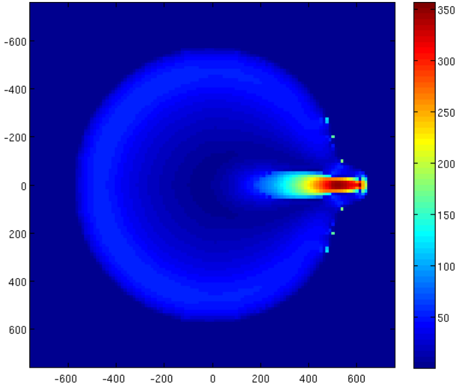
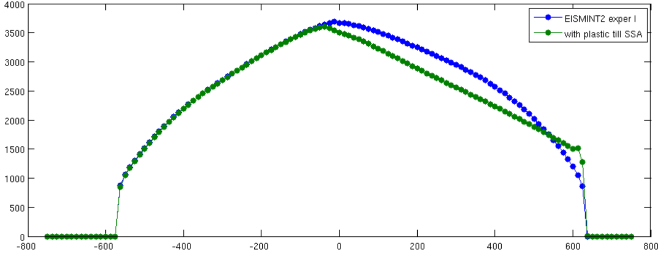
run for 5ka and look at velocity

use end values of EISMINT2 experiment I as starting state, on 12.5km grid, run for 5000 model years, and look at vertically-averaged horizontal velocity:

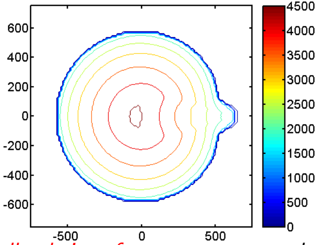


ice thickness



Above: ice thickness along centerline of trough (x-axis)

Right: surface elevation contours for plastic till SSA version

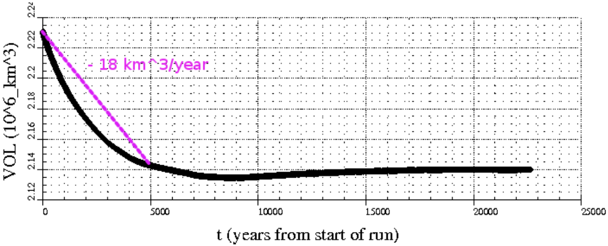


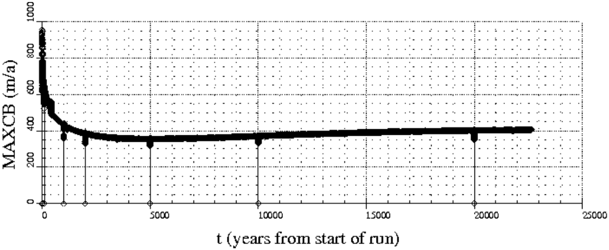
if surface elevation expression of ice streams was *really obvious from contour map* *then NE Greenland ice stream would have been discovered earlier!*

On time-stepping

- explicit
- diffusive vertical-plane-shear flow from SIA has a condition for stability (*typical: 0.3 year for 12.5 km grid*)
- sliding flow from SSA is upwinded
- sliding flow from SSA has another condition for stability (*typical: 0.1 year for 12.5 km grid*)
- plastic till model used as sliding law everywhere at base
- temperature equation solved every time step; *dirt cheap compared to SSA solution*

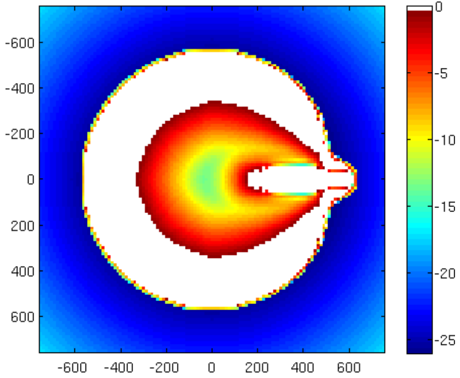
Longer run: 22ka (as of Tuesday)

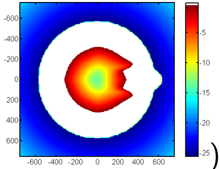




[EXPLAIN "MAXCB"]

basal homologous temperature (°C) at 5ka



(compare 

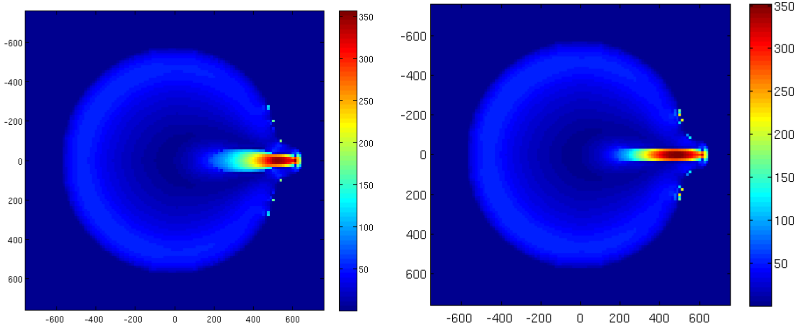
)

experiments: other parameter choices

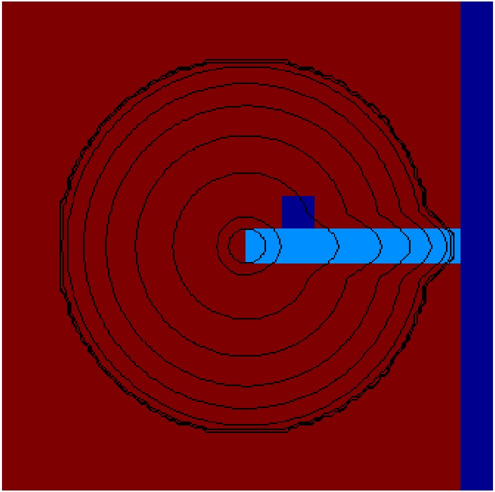
Exper.	Grid (hor,vert)	Duration	Stream width	Downstream ϕ	Lake?
P1	(12.5 km, 20 m)	5k a	100 km	5°	no
P2	(12.5 km, 20 m)	5k a	$\boxed{50 \text{ km}}$	5°	no
P3	(12.5 km, 20 m)	5k a	100 km	$\boxed{8^\circ}$	no
P4	(12.5 km, 20 m)	5k a	100 km	5°	$\boxed{\text{yes}}$
P5	(12.5 km, 20 m)	$\boxed{100\text{k a}}$	100 km	5°	no
P6	($\boxed{25 \text{ km}}$, 20 m)	5k a	100 km	5°	no
P7	($\boxed{7.5 \text{ km}}$, 20 m)	5k a	100 km	5°	no
P8	(12.5 km, $\boxed{10 \text{ m}}$)	5k a	100 km	5°	no

all start with final state of EISMINT II experiment I

ice stream widths of 100 km (P1) and 50 km (P2)

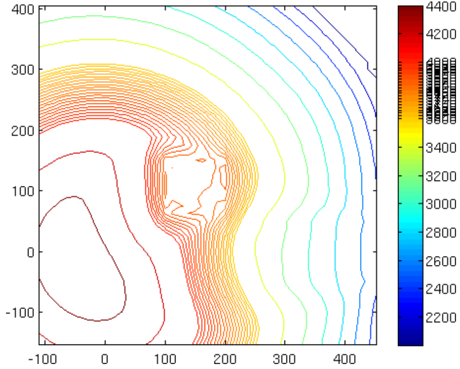


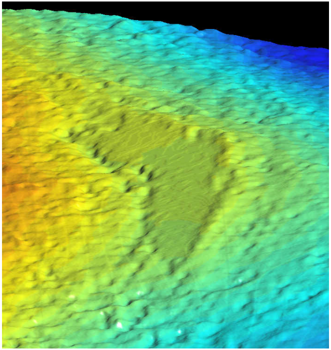
Experiment P4 adds "lake"



dark blue has till friction angle $\phi = 0^\circ$

"Lake" in experiment P4: surface elevation





Left: Detail of surface topography above $\theta = 0^\circ$ till "lake" in experiment P4. Contours every 20 m between 3600 m and 4000 m.

Right: a perspective view of the ice surface above Lake Vostok compiled from ERS-1 radar altimeter data provided by the National Snow and Ice Data Center. Lake Vostok is the flat, featureless area. From Michael Studinger's web page.

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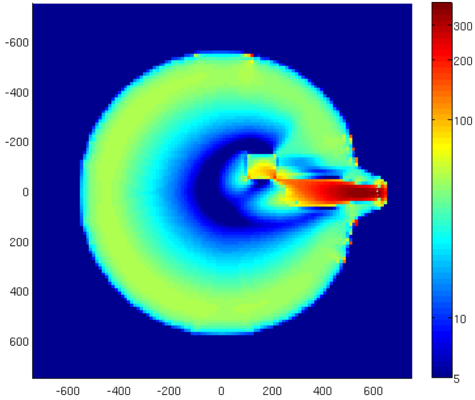
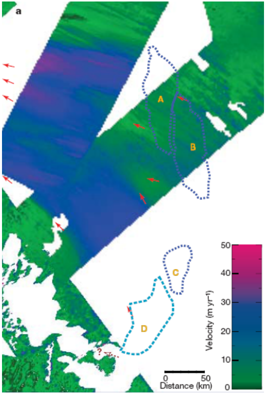
"Lake": vertically-averaged horizontal speed





Left: Vertically-averaged horizontal speed for P4 at 5ka.

Right: InSAR relative ice-surface velocity in m/a for the Recovery ice stream catchment. Red arrows show locations of clearly defined flow stripes. From Belle et al (2007) "Large subglacial lakes in East Antarctica at the onset of fast-flowing ice streams," *Nature*.

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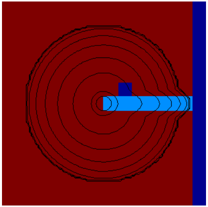
max velocity at end of run in experiments

Experiment	(description)	Max sliding speed at end of run (m/a)
P1	default	356
P2	narrower weak till strip	352
P3	stronger downstream till	164
P4	"lake"	389
P6	coarse (25 km) hor. grid	270
P7	fine (7.5 km) hor. grid	5301
P8	fine (10 m) vertical grid	358



[comment on bad (and revisable) experimental design!]

PISM = Parallel Ice Sheet Model

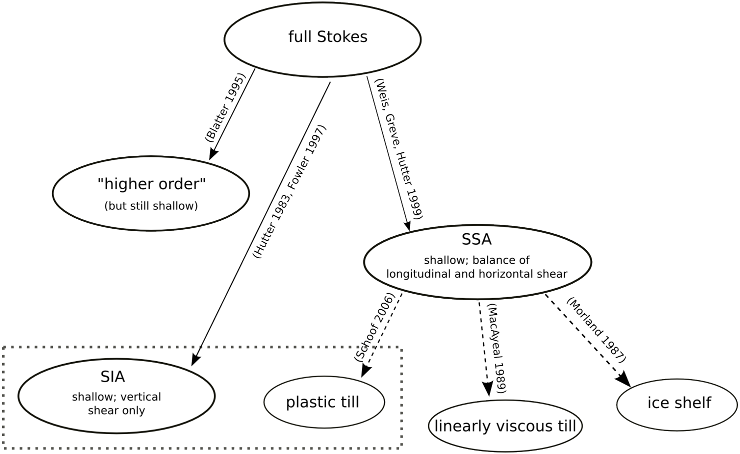
- open source:
  - website https://gna.org/projects/pism/
  - source via Subversion:

    `svn co http://svn.gna.org/svn/pism/trunk pism`
  - documentation at www.pism-docs.org
  - 75 page *User's Manual* includes how to perform verification (lots), EISMINT2, EISMINT-Ross, and EISMINT-Greenland
- under active development
- structurally parallel using PETSc and MPI; has run well on 500 processors

PDEs approximated by PISM

- map-plane conservation of mass
- incompressibility
- 3D (shallow approximation of) conservation of energy including bulk strain heating and friction heating from basal sliding and bedrock thermal model
- computation of basal melt water from conservation of energy; local (column-wise) conservation of melt water; freeze-on can occur
- earth deformation (by new, fast method)

And stress balance, of course



rectangle shows model for this talk

Stress balance equations for this talk

SIA:

$$(u_1, u_2) = -2(\rho g)^2 \, |\nabla h|^n \left[\int_b^z A(T^*) \, (h - \zeta)^n \right] \left(\frac{\partial h}{\partial x_1}, \frac{\partial h}{\partial x_2} \right)$$

SSA:

$$\frac{\partial}{\partial x_1} \left[2\overline{\nu} H \left(2 \frac{\partial v_1}{\partial x_1} + \frac{\partial v_2}{\partial x_2} \right) \right] + \frac{\partial}{\partial x_2} \left[\overline{\nu} H \left(\frac{\partial v_1}{\partial x_2} + \frac{\partial v_2}{\partial x_1} \right) \right] + \tau_{b,1} = \rho g H \frac{\partial h}{\partial x_1}$$

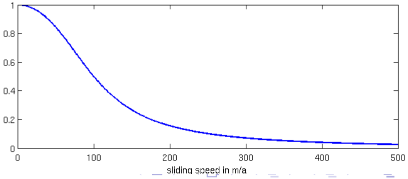
$$\frac{\partial}{\partial x_1} \left[\overline{\nu} H \left(\frac{\partial v_1}{\partial x_2} + \frac{\partial v_2}{\partial x_1} \right) \right] + \frac{\partial}{\partial x_2} \left[2\overline{\nu} H \left(\frac{\partial v_1}{\partial x_1} + 2 \frac{\partial v_2}{\partial x_2} \right) \right] + \tau_{b,2} = \rho g H \frac{\partial h}{\partial x_2}$$

with plastic till (Schoof, 2006)

$$\tau_b = -\tau_c \frac{(v_1, v_2)}{(v_1^2 + v_2^2)^{1/2}}$$

combined by "superposition":

$$(U_1, U_2) = f\left((v_1^2 + v_2^2)^{1/2} \right) \; \mathbf{u} + \mathbf{v}$$



Physics which is *not* in PISM

- full Stokes equations (i.e. *sans* shallowness assumptions)
- “Blatter model” (yet)
- true polythermal ice (yet)
- anisotropic flow laws

Verification and Validation

DEFINITION:

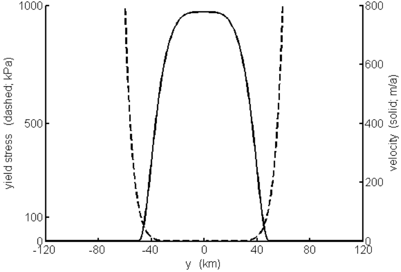
Verification is measuring the difference between numerical results and exact solutions and measuring the rate at which numerics converge to exact continuum values as grid is refined.

DEFINITION:

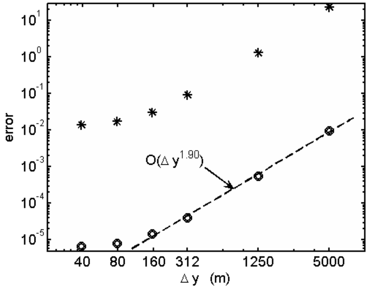
Validation is measuring the difference between model results and (trusted) observations of real systems and understanding what part of the difference is ascribable to flaws in the model.

Convergence for an exact solution of SSA with plastic till

Exact solution to "plastic till ice stream on a slope" appears in (Schoof, 2006).



solid: Velocity in the down slope direction; "y" is transverse. *dashed*: Till yield stress; grows sharply at $|y| = 40$ km.
Significant sliding occurs only in interval $|y| < 40$ km, but sliding region is not predetermined.

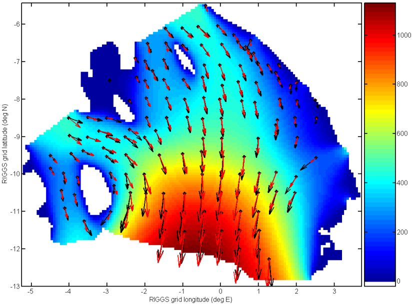


stars: maximum absolute velocity errors.
circles: average errors relative to maximum velocity.
Convergence at nearly optimal rate of $O(\Delta y^2)$.

Validation of PISM as an ice shelf flow model

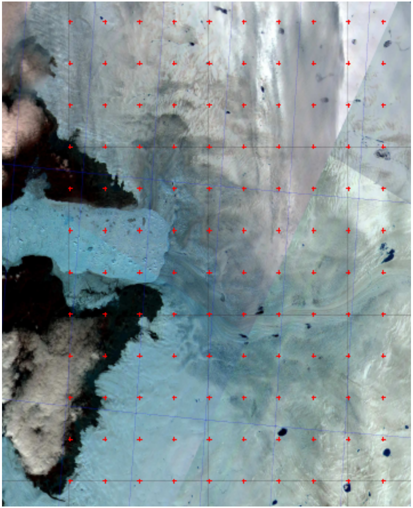
Color shows PISM's modeled speed (m/a) on Ross Ice Shelf with 6.8 km grid, from RIGGS/EISMINT-Ross data (MacAyeal and others, 1996).

- **black** arrows are observed velocities (RIGGS)
- **red** arrows are PISM model velocities; color is speed



note 6.8 km is fine enough to resolve geometry and ice stream/glacier inputs

Model resolution and outlet glaciers



red crosses show superimposed 5 km grid

Jakobshavns Ice Stream (west Greenland) figure courtesy of M. Truffer

Conclusions

- shallow model which can produce thermomechanically-coupled ice sheet *and* stream flow with a believable dynamical origin
- long runs (e.g. 20k models years in 500 processor hours on 12.5 km grid) at high resolution (12.5 km grid) with PISM
- it is ready to model Greenland ice sheet, especially if provided with latest basal shear stress estimates from inverse modeling
- marine ice sheets and grounding lines: more work needed (see MISMIP . . .)

Acknowledgements

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and also to Dave Covey *at GI and to* Don Bahls *at ARSC (for help making PISM work on the* `midnight` *Sun),*

and to NASA Cryospheric Sciences Program *for support from grant NAG5-11371.*